

# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "7 Inverse hyperbolic functions\7.2 Inverse hyperbolic cosine"

Test results for the 166 problems in "7.2.2 (d x)^m (a+b arccosh(c x))^n.m"

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^4}{x^2} dx$$

Optimal (type 4, 150 leaves, 11 steps):

$$\begin{aligned} & -\frac{\text{ArcCosh}[a x]^4}{x} + 8 a \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right] - 12 i a \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right] + \\ & 12 i a \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right] + 24 i a \text{ArcCosh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcCosh}[a x]}\right] - \\ & 24 i a \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right] - 24 i a \text{PolyLog}\left[4, -i e^{\text{ArcCosh}[a x]}\right] + 24 i a \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right] \end{aligned}$$

Result (type 4, 478 leaves):

$$\begin{aligned} a \left( -\frac{7 i \pi^4}{16} + \frac{1}{2} \pi^3 \text{ArcCosh}[a x] - \frac{3}{2} i \pi^2 \text{ArcCosh}[a x]^2 - 2 \pi \text{ArcCosh}[a x]^3 + i \text{ArcCosh}[a x]^4 - \frac{\text{ArcCosh}[a x]^4}{a x} + \frac{1}{2} \pi^3 \text{Log}\left[1 + i e^{-\text{ArcCosh}[a x]}\right] - \right. \\ \left. 3 i \pi^2 \text{ArcCosh}[a x] \text{Log}\left[1 + i e^{-\text{ArcCosh}[a x]}\right] - 6 \pi \text{ArcCosh}[a x]^2 \text{Log}\left[1 + i e^{-\text{ArcCosh}[a x]}\right] + 4 i \text{ArcCosh}[a x]^3 \text{Log}\left[1 + i e^{-\text{ArcCosh}[a x]}\right] + \right. \\ \left. 3 i \pi^2 \text{ArcCosh}[a x] \text{Log}\left[1 - i e^{\text{ArcCosh}[a x]}\right] + 6 \pi \text{ArcCosh}[a x]^2 \text{Log}\left[1 - i e^{\text{ArcCosh}[a x]}\right] - \frac{1}{2} \pi^3 \text{Log}\left[1 + i e^{\text{ArcCosh}[a x]}\right] - \right. \\ \left. 4 i \text{ArcCosh}[a x]^3 \text{Log}\left[1 + i e^{\text{ArcCosh}[a x]}\right] + \frac{1}{2} \pi^3 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (\pi + 2 i \text{ArcCosh}[a x])\right]\right] + 3 i (\pi - 2 i \text{ArcCosh}[a x])^2 \text{PolyLog}\left[2, -i e^{-\text{ArcCosh}[a x]}\right] - \right. \\ \left. 12 i \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right] + 3 i \pi^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right] + 12 \pi \text{ArcCosh}[a x] \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right] + \right. \\ \left. 12 \pi \text{PolyLog}\left[3, -i e^{-\text{ArcCosh}[a x]}\right] - 24 i \text{ArcCosh}[a x] \text{PolyLog}\left[3, -i e^{-\text{ArcCosh}[a x]}\right] + 24 i \text{ArcCosh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcCosh}[a x]}\right] - \right. \\ \left. 12 \pi \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right] - 24 i \text{PolyLog}\left[4, -i e^{-\text{ArcCosh}[a x]}\right] - 24 i \text{PolyLog}\left[4, -i e^{\text{ArcCosh}[a x]}\right] \right) \end{aligned}$$

### Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^4}{x^4} dx$$

Optimal (type 4, 268 leaves, 19 steps):

$$\begin{aligned} & \frac{2 a^2 \text{ArcCosh}[a x]^2}{x} + \frac{2 a \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^3}{3 x^2} - \frac{\text{ArcCosh}[a x]^4}{3 x^3} - 8 a^3 \text{ArcCosh}[a x] \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right] + \\ & \frac{4}{3} a^3 \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right] + 4 i a^3 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right] - 2 i a^3 \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right] - \\ & 4 i a^3 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right] + 2 i a^3 \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right] + 4 i a^3 \text{ArcCosh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcCosh}[a x]}\right] - \\ & 4 i a^3 \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right] - 4 i a^3 \text{PolyLog}\left[4, -i e^{\text{ArcCosh}[a x]}\right] + 4 i a^3 \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right] \end{aligned}$$

Result (type 4, 595 leaves):

$$\begin{aligned}
& a^3 \left( \frac{1}{2} i \left( 8 + \pi^2 - 4 i \pi \operatorname{ArcCosh}[a x] - 4 \operatorname{ArcCosh}[a x]^2 \right) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[a x]}\right] - \right. \\
& \frac{1}{96} i \left( 7 \pi^4 + 8 i \pi^3 \operatorname{ArcCosh}[a x] + 24 \pi^2 \operatorname{ArcCosh}[a x]^2 + \frac{192 i \operatorname{ArcCosh}[a x]^2}{a x} - 32 i \pi \operatorname{ArcCosh}[a x]^3 + \right. \\
& \frac{64 i \sqrt{\frac{-1+ax}{1+ax}} (1+ax) \operatorname{ArcCosh}[a x]^3}{a^2 x^2} - 16 \operatorname{ArcCosh}[a x]^4 - \frac{32 i \operatorname{ArcCosh}[a x]^4}{a^3 x^3} - 384 \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[a x]}\right] + \\
& 8 i \pi^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[a x]}\right] + 384 \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[a x]}\right] + 48 \pi^2 \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[a x]}\right] - \\
& 96 i \pi \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[a x]}\right] - 64 \operatorname{ArcCosh}[a x]^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[a x]}\right] - 48 \pi^2 \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 - i e^{\operatorname{ArcCosh}[a x]}\right] + \\
& 96 i \pi \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - i e^{\operatorname{ArcCosh}[a x]}\right] - 8 i \pi^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcCosh}[a x]}\right] + 64 \operatorname{ArcCosh}[a x]^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcCosh}[a x]}\right] + \\
& 8 i \pi^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcCosh}[a x])\right]\right] + 384 \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[a x]}\right] + 192 \operatorname{ArcCosh}[a x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[a x]}\right] - \\
& 48 \pi^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[a x]}\right] + 192 i \pi \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[a x]}\right] + 192 i \pi \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[a x]}\right] + \\
& 384 \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[a x]}\right] - 384 \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[a x]}\right] - \\
& \left. \left. \left. 192 i \pi \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[a x]}\right] + 384 \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcCosh}[a x]}\right] + 384 \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[a x]}\right] \right) \right) \right)
\end{aligned}$$

Problem 117: Unable to integrate problem.

$$\int x^m \operatorname{ArcCosh}[a x]^2 dx$$

Optimal (type 5, 154 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{ArcCosh}[a x]^2}{1+m} - \frac{2 a x^{2+m} \sqrt{1-a x} \operatorname{ArcCosh}[a x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2+3m+m^2) \sqrt{-1+ax}} - \frac{2 a^2 x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right\}, a^2 x^2\right]}{6+11m+6m^2+m^3}$$

Result (type 8, 12 leaves):

$$\int x^m \operatorname{ArcCosh}[a x]^2 dx$$

**Problem 118:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m \operatorname{ArcCosh}[a x] dx$$

Optimal (type 5, 91 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{ArcCosh}[a x]}{1+m} - \frac{a x^{2+m} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2+3m+m^2) \sqrt{-1+ax} \sqrt{1+ax}}$$

Result (type 6, 329 leaves):

$$\frac{1}{1+m} x^m \left( - \left( \left( 12 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-ax, \frac{1}{2}(1-ax)\right] \right) / \left( a \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-ax, \frac{1}{2}(1-ax)\right] + (-1+ax) \right. \right. \right. \right. \right. \\ \left. \left. \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-ax, \frac{1}{2}(1-ax)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-ax, \frac{1}{2}(1-ax)\right] \right) \right) \right) + \\ \left( 12 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-ax, \frac{1}{2}(1-ax)\right] \right) / \left( a \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-ax, \frac{1}{2}(1-ax)\right] + \right. \right. \\ \left. \left. (-1+ax) \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-ax, \frac{1}{2}(1-ax)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-ax, \frac{1}{2}(1-ax)\right] \right) \right) \right) + x \operatorname{ArcCosh}[a x] \right)$$

**Problem 163:** Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{fx} (a + b \operatorname{ArcCosh}[cx])^2 dx$$

Optimal (type 5, 128 leaves, 2 steps):

$$\frac{2 (f x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 f} - \frac{8 b c (f x)^{5/2} \sqrt{1 - c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{15 f^2 \sqrt{-1 + c x}} - \frac{16 b^2 c^2 (f x)^{7/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{105 f^3}$$

Result (type 5, 256 leaves):

$$\frac{1}{27} \sqrt{f x} \left( \begin{aligned} & 18 a^2 x + 36 a b x \operatorname{ArcCosh}[c x] - \frac{24 b^2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{c} + \\ & 2 b^2 x (8 + 9 \operatorname{ArcCosh}[c x]^2) - \frac{24 a b \left( \sqrt{-1+c x} (1+c x) + \frac{i \sqrt{\frac{1+c x}{-1+c x}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+c x}}\right], 2\right]}{\sqrt{\frac{c x}{-1+c x}}} \right)}{c \sqrt{1+c x}} + \\ & \frac{24 b^2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, c^2 x^2\right]}{c} - \frac{3 \sqrt{2} b^2 \pi x \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, c^2 x^2\right]}{\Gamma\left[\frac{5}{4}\right] \Gamma\left[\frac{7}{4}\right]} \end{aligned} \right)$$

**Problem 164: Unable to integrate problem.**

$$\int (d x)^m (a + b \operatorname{ArcCosh}[c x])^2 dx$$

Optimal (type 5, 181 leaves, 2 steps):

$$\frac{(dx)^{1+m} (a + b \operatorname{ArcCosh}[cx])^2}{d(1+m)} - \frac{2bc(dx)^{2+m} \sqrt{1-cx} (a + b \operatorname{ArcCosh}[cx]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{d^2(1+m)(2+m)\sqrt{-1+cx}}$$

$$\frac{2b^2 c^2 (dx)^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right\}, c^2 x^2\right]}{d^3(1+m)(2+m)(3+m)}$$

Result (type 8, 18 leaves):

$$\int (dx)^m (a + b \operatorname{ArcCosh}[cx])^2 dx$$

**Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (dx)^m (a + b \operatorname{ArcCosh}[cx]) dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$\frac{(dx)^{1+m} (a + b \operatorname{ArcCosh}[cx])}{d(1+m)} - \frac{bc(dx)^{2+m} \sqrt{1-c^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{d^2(1+m)(2+m)\sqrt{-1+cx}\sqrt{1+cx}}$$

Result (type 6, 337 leaves):

$$\frac{1}{1+m} (dx)^m$$

$$\left( - \left( \left( 12b \sqrt{-1+cx} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( c \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + (-1+cx) \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \right) \right) \right) + \\ \left( 12b \sqrt{\frac{-1+cx}{1+cx}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( c \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \right. \\ \left. \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \right) \right) + x (a + b \operatorname{ArcCosh}[cx]) \right)$$

## Test results for the 569 problems in "7.2.4 (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.m"

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x (d - c^2 d x^2)} dx$$

Optimal (type 4, 61 leaves, 7 steps):

$$\frac{2 (a + b \operatorname{ArcCosh}[c x]) \operatorname{ArcTanh}\left[e^{2 \operatorname{ArcCosh}[c x]}\right]}{d} + \frac{b \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[c x]}\right]}{2 d} - \frac{b \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[c x]}\right]}{2 d}$$

Result (type 4, 124 leaves):

$$-\frac{1}{2 d} \left( -2 b \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right] + 2 b \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - e^{-\operatorname{ArcCosh}[c x]}\right] + 2 b \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + e^{-\operatorname{ArcCosh}[c x]}\right] - 2 a \operatorname{Log}[x] + a \operatorname{Log}\left[1 - c^2 x^2\right] + b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right] - 2 b \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcCosh}[c x]}\right] - 2 b \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcCosh}[c x]}\right] \right)$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x (d - c^2 d x^2)^2} dx$$

Optimal (type 4, 116 leaves, 9 steps):

$$-\frac{b c x}{2 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{a + b \operatorname{ArcCosh}[c x]}{2 d^2 (1 - c^2 x^2)} + \frac{2 (a + b \operatorname{ArcCosh}[c x]) \operatorname{ArcTanh}\left[e^{2 \operatorname{ArcCosh}[c x]}\right]}{d^2} + \frac{b \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[c x]}\right]}{2 d^2} - \frac{b \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[c x]}\right]}{2 d^2}$$

Result (type 4, 243 leaves):

$$\frac{1}{4 d^2} \left( -b \sqrt{\frac{-1+c x}{1+c x}} + \frac{b \sqrt{\frac{-1+c x}{1+c x}}}{1-c x} + \frac{b c x \sqrt{\frac{-1+c x}{1+c x}}}{1-c x} - \frac{2 a}{-1+c^2 x^2} + \frac{b \operatorname{ArcCosh}[c x]}{1-c x} + \frac{b \operatorname{ArcCosh}[c x]}{1+c x} + \right. \\ \left. 4 b \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c x]}\right] - 4 b \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1-e^{-\operatorname{ArcCosh}[c x]}\right] - 4 b \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1+e^{-\operatorname{ArcCosh}[c x]}\right] + \right. \\ \left. 4 a \operatorname{Log}[x] - 2 a \operatorname{Log}\left[1-c^2 x^2\right] - 2 b \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcCosh}[c x]}\right] + 4 b \operatorname{PolyLog}\left[2,-e^{-\operatorname{ArcCosh}[c x]}\right] + 4 b \operatorname{PolyLog}\left[2,e^{-\operatorname{ArcCosh}[c x]}\right] \right)$$

**Problem 119: Unable to integrate problem.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 84 leaves, 2 steps):

$$\frac{x (a + b \operatorname{ArcCosh}[c x])}{d \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1+c x} \sqrt{1+c x} \operatorname{Log}[1 - c^2 x^2]}{2 c d \sqrt{d - c^2 d x^2}}$$

Result (type 8, 26 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d - c^2 d x^2)^{3/2}} dx$$

**Problem 121: Unable to integrate problem.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 158 leaves, 5 steps):

$$-\frac{a + b \operatorname{ArcCosh}[c x]}{d x \sqrt{d - c^2 d x^2}} + \frac{2 c^2 x (a + b \operatorname{ArcCosh}[c x])}{d \sqrt{d - c^2 d x^2}} + \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{d^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{2 d^2 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 8, 29 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 (d - c^2 d x^2)^{3/2}} dx$$



### Problem 123: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$\begin{aligned} & -\frac{b c \sqrt{d - c^2 d x^2}}{6 d^2 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{a + b \operatorname{ArcCosh}[c x]}{3 d x^3 \sqrt{d - c^2 d x^2}} - \frac{4 c^2 (a + b \operatorname{ArcCosh}[c x])}{3 d x \sqrt{d - c^2 d x^2}} + \\ & \frac{8 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d \sqrt{d - c^2 d x^2}} + \frac{5 b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{2 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d - c^2 d x^2)^{3/2}} dx$$

### Problem 127: Unable to integrate problem.

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\frac{b \sqrt{-1 + c x} \sqrt{1 + c x}}{6 c^3 d (d - c^2 d x^2)^{3/2}} + \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{6 c^3 d^2 \sqrt{d - c^2 d x^2}}$$

Result (type 8, 29 leaves):

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

### Problem 129: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 162 leaves, 5 steps):

$$\frac{b \sqrt{-1 + c x} \sqrt{1 + c x}}{6 c d (d - c^2 d x^2)^{3/2}} + \frac{x (a + b \operatorname{ArcCosh}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{2 x (a + b \operatorname{ArcCosh}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{3 c d^2 \sqrt{d - c^2 d x^2}}$$

Result (type 8, 26 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d - c^2 d x^2)^{5/2}} dx$$

Problem 131: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 (d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 248 leaves, 5 steps):

$$\begin{aligned} & - \frac{b c \sqrt{d - c^2 d x^2}}{6 d^3 \sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)} - \frac{a + b \operatorname{ArcCosh}[c x]}{d x (d - c^2 d x^2)^{3/2}} + \frac{4 c^2 x (a + b \operatorname{ArcCosh}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \\ & \frac{8 c^2 x (a + b \operatorname{ArcCosh}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 b c \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{6 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 (d - c^2 d x^2)^{5/2}} dx$$

Problem 133: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 338 leaves, 5 steps):

$$\begin{aligned} & - \frac{b c \sqrt{d - c^2 d x^2}}{6 d^3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^3 \sqrt{d - c^2 d x^2}}{6 d^3 \sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)} - \frac{a + b \operatorname{ArcCosh}[c x]}{3 d x^3 (d - c^2 d x^2)^{3/2}} - \frac{2 c^2 (a + b \operatorname{ArcCosh}[c x])}{d x (d - c^2 d x^2)^{3/2}} + \\ & \frac{8 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{16 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{8 b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{3 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{3 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d - c^2 d x^2)^{5/2}} dx$$

**Problem 143:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(f x)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 5, 98 leaves, 1 step):

$$\frac{2 (f x)^{5/2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{5 f} + \frac{4 b c (f x)^{7/2} \sqrt{-1 + c x} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{35 f^2 \sqrt{1 - c x}}$$

Result (type 5, 230 leaves):

$$\frac{1}{36 c^2 \sqrt{1 - c^2 x^2}} f \sqrt{f x} \left( \frac{24 i a \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{x} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right], -1\right]}{\sqrt{-\frac{1}{c}}} + \right.$$

$$8 (1 + c x) \left( -3 a + 3 a c x - 2 b c x \sqrt{\frac{-1 + c x}{1 + c x}} + 3 b (-1 + c x) \operatorname{ArcCosh}[c x] - 3 b (-1 + c x) \operatorname{ArcCosh}[c x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, c^2 x^2\right] \right) +$$

$$\left. \frac{3 \sqrt{2} b c \pi x \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, c^2 x^2\right]}{\Gamma\left[\frac{5}{4}\right] \Gamma\left[\frac{7}{4}\right]} \right)$$

**Problem 144:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f x)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 5, 141 leaves, 1 step):

$$\frac{2 (f x)^{5/2} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{5 f \sqrt{d - c^2 d x^2}} +$$

$$\frac{4 b c (f x)^{7/2} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{35 f^2 \sqrt{d - c^2 d x^2}}$$

Result (type 5, 241 leaves):

$$\frac{1}{36 c^2 \sqrt{d - c^2 d x^2} \text{Gamma}\left[\frac{5}{4}\right] \text{Gamma}\left[\frac{7}{4}\right]}$$

$$f \sqrt{f x} \left( 8 \text{Gamma}\left[\frac{5}{4}\right] \text{Gamma}\left[\frac{7}{4}\right] \left( \frac{3 i a \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right], -1\right]}{\sqrt{-\frac{1}{c}}} + (1 + c x) \left( -3 a + 3 a c x - \right. \right. \right.$$

$$\left. \left. 2 b c x \sqrt{\frac{-1 + c x}{1 + c x}} + 3 b (-1 + c x) \text{ArcCosh}[c x] - 3 b (-1 + c x) \text{ArcCosh}[c x] \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, c^2 x^2\right] \right) \right) +$$

$$\left. \left. 3 \sqrt{2} b c \pi x \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \text{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, c^2 x^2\right] \right) \right)$$

**Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (f x)^m (d - c^2 d x^2)^3 (a + b \text{ArcCosh}[c x]) dx$$

Optimal (type 5, 429 leaves, 8 steps):

$$-\frac{b c d^3 (2271 + 1329 m + 284 m^2 + 27 m^3 + m^4) (f x)^{2+m} (1 - c^2 x^2)}{f^2 (3 + m)^2 (5 + m)^2 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d^3 (9 + m) (13 + 2 m) (f x)^{4+m} (1 - c^2 x^2)}{f^4 (5 + m)^2 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^3 (f x)^{6+m} (1 - c^2 x^2)}{f^6 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} +$$

$$\frac{d^3 (f x)^{1+m} (a + b \text{ArcCosh}[c x])}{f (1 + m)} - \frac{3 c^2 d^3 (f x)^{3+m} (a + b \text{ArcCosh}[c x])}{f^3 (3 + m)} + \frac{3 c^4 d^3 (f x)^{5+m} (a + b \text{ArcCosh}[c x])}{f^5 (5 + m)} -$$

$$\frac{c^6 d^3 (f x)^{7+m} (a + b \text{ArcCosh}[c x])}{f^7 (7 + m)} - \frac{3 b c d^3 (2161 + 1813 m + 455 m^2 + 35 m^3) (f x)^{2+m} \sqrt{1 - c^2 x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{f^2 (1 + m) (2 + m) (3 + m)^2 (5 + m)^2 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 6, 3439 leaves):

$$\begin{aligned}
& \frac{a d^3 x (f x)^m}{1+m} - \frac{3 a c^2 d^3 x^3 (f x)^m}{3+m} + \frac{3 a c^4 d^3 x^5 (f x)^m}{5+m} - \frac{a c^6 d^3 x^7 (f x)^m}{7+m} + \frac{1}{c} b d^3 (c x)^{-m} (f x)^m \\
& \left( -\frac{1}{1+m} 12 (c x)^m \left( \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) - \right. \\
& \quad \left. \left( \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \right. \right. \\
& \quad \left. \left. \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \frac{(c x)^{1+m} \operatorname{ArcCosh}[c x]}{1+m} \Bigg) - \\
& 3 b c d^3 x^2 (c x)^{-2-m} (f x)^m \left( -\frac{1}{3+m} 4 (c x)^m \left( \left( 3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \right. \right. \\
& \quad \left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \quad \left( 3 \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) + \\
& \quad (-1+c x)^{3/2} \sqrt{1+c x} \left( \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + 3 \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \\
& \quad \left( 7 (-1+c x) \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + 5 \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{(c x)^{3+m} \text{ArcCosh}[c x]}{3+m} \right) + 3 b c^3 d^3 x^4 (c x)^{-4-m} (f x)^m \left( -\frac{1}{5+m} \left( \left( 12 (c x)^m \sqrt{-1+c x} \sqrt{1+c x} \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \right. \right. \\
& \left. \left( 6 \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \left. \left. (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \text{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \left( 12 (c x)^m \sqrt{\frac{-1+c x}{1+c x}} \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \left. 4 m (-1+c x) \text{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - (-1+c x) \text{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \\
& \left( 40 (c x)^m (-1+c x)^{3/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \text{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \left. 3 (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \text{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) + \\
& \left( 112 (c x)^m (-1+c x)^{5/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \text{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \left. 5 (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \text{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) + \\
& \left( 108 (c x)^m (-1+c x)^{7/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 7 \left( 18 \text{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \left. \left. (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{9}{2}, 1-m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \text{AppellF1}\left[\frac{9}{2}, -m, \frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \\
& \left( 44 (c x)^m (-1+c x)^{9/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 9 \left( 22 \text{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{11}{2}, 1-m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{11}{2}, -m, \frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) \right) + \frac{(c x)^{5+m} \text{ArcCosh}[c x]}{5+m} \left. \right) - b c^5 d^3 x^6 (c x)^{-6-m} (f x)^m \\
& \left( -\frac{1}{7+m} \left( \left( 12 (c x)^m \sqrt{-1+c x} \sqrt{1+c x} \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \right. \\
& \left. \left. (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \text{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 12 (c x)^m \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \\
& 4 m (-1+c x) \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - (-1+c x) \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \Big) + \\
& \left( 60 (c x)^m (-1+c x)^{3/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \\
& 3 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \Big) + \\
& \left( 252 (c x)^m (-1+c x)^{5/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \\
& 5 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \Big) + \\
& \left( 468 (c x)^m (-1+c x)^{7/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 7 \left( 18 \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \right. \\
& \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{9}{2}, 1-m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{9}{2}, -m, \frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Big) + \\
& \left( 484 (c x)^m (-1+c x)^{9/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 9 \left( 22 \operatorname{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \right. \\
& \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{11}{2}, 1-m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{11}{2}, -m, \frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Big) + \\
& \left( 260 (c x)^m (-1+c x)^{11/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{11}{2}, -m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 11 \left( 26 \operatorname{AppellF1}\left[\frac{11}{2}, -m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \right. \\
& \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{13}{2}, 1-m, -\frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{13}{2}, -m, \frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Big) + \\
& \left( 60 (c x)^m (-1+c x)^{13/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{13}{2}, -m, -\frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 13 \left( 30 \operatorname{AppellF1}\left[\frac{13}{2}, -m, -\frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{15}{2}, 1-m, -\frac{1}{2}, \frac{17}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{15}{2}, -m, \frac{1}{2}, \frac{17}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Big) + \frac{(c x)^{7+m} \operatorname{ArcCosh}[c x]}{7+m}
\end{aligned}$$

**Problem 146: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (f x)^m (d - c^2 d x^2)^2 (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 307 leaves, 7 steps):

$$\frac{b c d^2 (38 + 13 m + m^2) (f x)^{2+m} (1 - c^2 x^2)}{f^2 (3 + m)^2 (5 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d^2 (f x)^{4+m} (1 - c^2 x^2)}{f^4 (5 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{d^2 (f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{f (1 + m)} - \frac{2 c^2 d^2 (f x)^{3+m} (a + b \operatorname{ArcCosh}[c x])}{f^3 (3 + m)} +$$

$$\frac{c^4 d^2 (f x)^{5+m} (a + b \operatorname{ArcCosh}[c x])}{f^5 (5 + m)} - \frac{b c d^2 (149 + 100 m + 15 m^2) (f x)^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{f^2 (1 + m) (2 + m) (3 + m)^2 (5 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 6, 2085 leaves):

$$\frac{a d^2 x (f x)^m}{1 + m} - \frac{2 a c^2 d^2 x^3 (f x)^m}{3 + m} + \frac{a c^4 d^2 x^5 (f x)^m}{5 + m} + \frac{1}{c} b d^2 (c x)^{-m} (f x)^m$$

$$\left( -\frac{1}{1 + m} 12 (c x)^m \left( \left( \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) + \right.$$

$$\left. (-1 + c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1 - m, -\frac{1}{2}, \frac{5}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) \right) -$$

$$\left( \sqrt{\frac{-1 + c x}{1 + c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] + (-1 + c x) \right.$$

$$\left. \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1 - m, \frac{1}{2}, \frac{5}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) \right) + \frac{(c x)^{1+m} \operatorname{ArcCosh}[c x]}{1 + m} \right) -$$

$$2 b c d^2 x^2 (c x)^{-2-m} (f x)^m \left( -\frac{1}{3 + m} 4 (c x)^m \left( \left( 3 \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) / \right.$$

$$\left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] + \right.$$

$$\left. (-1 + c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1 - m, -\frac{1}{2}, \frac{5}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1 - c x, \frac{1}{2} (1 - c x)\right] \right) \right) \right) -$$



$$\begin{aligned}
& \left( 3 \sqrt{\frac{-1+cx}{1+cx}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) + \\
& \quad (-1+cx)^{3/2} \sqrt{1+cx} \left( \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + 3 \right. \right. \\
& \quad \left. \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \right) + \\
& \quad \left( 7(-1+cx) \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + 5 \right. \\
& \quad \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \left. \right) \\
& \left. \frac{(cx)^{3+m} \operatorname{ArcCosh}[cx]}{3+m} \right) + bc^3 d^2 x^4 (cx)^{-4-m} (fx)^m \left( -\frac{1}{5+m} \left( \left( 12 (cx)^m \sqrt{-1+cx} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \right. \right. \\
& \quad \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \left. \right) - \\
& \quad \left( 12 (cx)^m \sqrt{\frac{-1+cx}{1+cx}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 4m (-1+cx) \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] - (-1+cx) \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) + \\
& \quad \left( 40 (cx)^m (-1+cx)^{3/2} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 3(-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) + \\
& \quad \left( 112 (cx)^m (-1+cx)^{5/2} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 5(-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) + \\
& \quad \left( 108 (cx)^m (-1+cx)^{7/2} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \\
& \quad \left( 7 \left( 18 \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned} & (-1 + cx) \left( 4m \operatorname{AppellF1} \left[ \frac{9}{2}, 1-m, -\frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx) \right] + \operatorname{AppellF1} \left[ \frac{9}{2}, -m, \frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx) \right] \right) + \\ & \left( 44 (cx)^m (-1+cx)^{9/2} \sqrt{1+cx} \operatorname{AppellF1} \left[ \frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx) \right] \right) / \\ & \left( 9 \left( 22 \operatorname{AppellF1} \left[ \frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx) \right] + (-1+cx) \left( 4m \operatorname{AppellF1} \left[ \frac{11}{2}, 1-m, -\frac{1}{2}, \frac{13}{2}, 1-cx, \frac{1}{2}(1-cx) \right] + \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{AppellF1} \left[ \frac{11}{2}, -m, \frac{1}{2}, \frac{13}{2}, 1-cx, \frac{1}{2}(1-cx) \right] \right) \right) \right) + \frac{(cx)^{5+m} \operatorname{ArcCosh}[cx]}{5+m} \end{aligned}$$

**Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (fx)^m (d - c^2 dx^2) (a + b \operatorname{ArcCosh}[cx]) dx$$

Optimal (type 5, 184 leaves, 6 steps):

$$\begin{aligned} & \frac{bcd (fx)^{2+m} \sqrt{-1+cx} \sqrt{1+cx}}{f^2 (3+m)^2} + \frac{d (fx)^{1+m} (a + b \operatorname{ArcCosh}[cx])}{f (1+m)} - \\ & \frac{c^2 d (fx)^{3+m} (a + b \operatorname{ArcCosh}[cx])}{f^3 (3+m)} - \frac{bcd (7+3m) (fx)^{2+m} \sqrt{1-c^2 x^2} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2 \right]}{f^2 (1+m) (2+m) (3+m)^2 \sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

Result (type 6, 1047 leaves):

$$\begin{aligned}
& \frac{a d x (f x)^m}{1+m} - \frac{a c^2 d x^3 (f x)^m}{3+m} + \frac{1}{c} b d (c x)^{-m} (f x)^m \\
& \left( -\frac{1}{1+m} 12 (c x)^m \left( \left( \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \quad \left( \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \right. \\
& \quad \left. \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \left. \right) + \frac{(c x)^{1+m} \operatorname{ArcCosh}[c x]}{1+m} \Bigg) - \\
& b c d x^2 (c x)^{-2-m} (f x)^m \left( -\frac{1}{3+m} 4 (c x)^m \left( \left( 3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \right. \right. \\
& \quad \left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \quad \left( 3 \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) + \\
& \quad (-1+c x)^{3/2} \sqrt{1+c x} \left( \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + 3 \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \left( 7 (-1+c x) \right. \\
& \quad \left. \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + 5 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) \left. \right) + \frac{(c x)^{3+m} \operatorname{ArcCosh}[c x]}{3+m} \Bigg)
\end{aligned}$$

### Problem 151: Unable to integrate problem.

$$\int (f x)^m (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 723 leaves, 11 steps):

$$\begin{aligned} & - \frac{b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2}}{f^2 (2+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{15 b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2}}{f^2 (2+m)^2 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} - \\ & \frac{5 b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2}}{f^2 (2+m) (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5 b c^3 d^2 (f x)^{4+m} \sqrt{d - c^2 d x^2}}{f^4 (4+m)^2 (6+m) \sqrt{-1+c x} \sqrt{1+c x}} + \frac{2 b c^3 d^2 (f x)^{4+m} \sqrt{d - c^2 d x^2}}{f^4 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} - \\ & \frac{b c^5 d^2 (f x)^{6+m} \sqrt{d - c^2 d x^2}}{f^6 (6+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{15 d^2 (f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f (6+m) (8+6m+m^2)} + \frac{5 d (f x)^{1+m} (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{f (4+m) (6+m)} + \\ & \frac{(f x)^{1+m} (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{f (6+m)} + \frac{15 d^2 (f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (4+m) (6+m) (2+3m+m^2) \sqrt{1-c x} \sqrt{1+c x}} - \\ & \frac{15 b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} \end{aligned}$$

Result (type 8, 31 leaves):

$$\int (f x)^m (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

### Problem 152: Unable to integrate problem.

$$\int (f x)^m (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 455 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 b c d (f x)^{2+m} \sqrt{d-c^2 d x^2}}{f^2 (2+m)^2 (4+m) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c d (f x)^{2+m} \sqrt{d-c^2 d x^2}}{f^2 (2+m) (4+m) \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{b c^3 d (f x)^{4+m} \sqrt{d-c^2 d x^2}}{f^4 (4+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{3 d (f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{f (8+6 m+m^2)} + \frac{(f x)^{1+m} (d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x])}{f (4+m)} + \\
& \frac{3 d (f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (4+m) (2+3 m+m^2) \sqrt{1-c x} \sqrt{1+c x}} - \\
& \frac{3 b c d (f x)^{2+m} \sqrt{d-c^2 d x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 (4+m) \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int (f x)^m (d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x]) dx$$

Problem 153: Unable to integrate problem.

$$\int (f x)^m \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 278 leaves, 3 steps):

$$\begin{aligned}
& - \frac{b c (f x)^{2+m} \sqrt{d-c^2 d x^2}}{f^2 (2+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{(f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{f (2+m)} + \\
& \frac{(f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (2+3 m+m^2) \sqrt{1-c x} \sqrt{1+c x}} - \\
& \frac{b c (f x)^{2+m} \sqrt{d-c^2 d x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int (f x)^m \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) dx$$

Problem 154: Unable to integrate problem.

$$\int \frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 5, 176 leaves, 1 step):

$$\frac{(f x)^{1+m} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f(1+m) \sqrt{d-c^2 d x^2}} +$$

$$\frac{b c (f x)^{2+m} \sqrt{-1+c x} \sqrt{1+c x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right\}, c^2 x^2\right]}{f^2(1+m)(2+m) \sqrt{d-c^2 d x^2}}$$

Result (type 9, 202 leaves):

$$-\frac{1}{(1+m) \sqrt{d-c^2 d x^2}} 2^{-2-m} x (f x)^m$$

$$\left( -2^{2+m} \left( a \sqrt{1-c^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] + b (1-c^2 x^2) \operatorname{ArcCosh}[c x] \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) - \right.$$

$$\left. b c (1+m) \sqrt{\pi} x \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{Gamma}[1+m] \operatorname{HypergeometricPFQRegularized}\left[\left\{1, \frac{2+m}{2}, \frac{2+m}{2}\right\}, \left\{\frac{3+m}{2}, \frac{4+m}{2}\right\}, c^2 x^2\right] \right)$$

Problem 155: Unable to integrate problem.

$$\int \frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])}{(d-c^2 d x^2)^{3/2}} dx$$

Optimal (type 5, 300 leaves, 4 steps):

$$\frac{(f x)^{1+m} (a+b \operatorname{ArcCosh}[c x])}{d f \sqrt{d-c^2 d x^2}} - \frac{m (f x)^{1+m} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{d f (1+m) \sqrt{d-c^2 d x^2}} +$$

$$\frac{b c (f x)^{2+m} \sqrt{-1+c x} \sqrt{1+c x} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{d f^2 (2+m) \sqrt{d-c^2 d x^2}} -$$

$$\frac{b c m (f x)^{2+m} \sqrt{-1+c x} \sqrt{1+c x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right\}, c^2 x^2\right]}{d f^2 (1+m)(2+m) \sqrt{d-c^2 d x^2}}$$

Result (type 8, 31 leaves):

$$\int \frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])}{(d-c^2 d x^2)^{3/2}} dx$$

### Problem 156: Unable to integrate problem.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 5, 450 leaves, 7 steps):

$$\begin{aligned} & \frac{(f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{3 d f (d - c^2 d x^2)^{3/2}} + \frac{(2 - m) (f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{3 d^2 f \sqrt{d - c^2 d x^2}} - \\ & \frac{(2 - m) m (f x)^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{3 d^2 f (1 + m) \sqrt{d - c^2 d x^2}} + \\ & \frac{b c (2 - m) (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d^2 f^2 (2 + m) \sqrt{d - c^2 d x^2}} + \\ & \frac{b c (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d^2 f^2 (2 + m) \sqrt{d - c^2 d x^2}} - \\ & \frac{b c (2 - m) m (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{3 d^2 f^2 (1 + m) (2 + m) \sqrt{d - c^2 d x^2}} \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

### Problem 157: Unable to integrate problem.

$$\int (f x)^m (d1 + c d1 x)^{5/2} (d2 - c d2 x)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 817 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b c d_1^2 d_2^2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^2 (2+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{15 b c d_1^2 d_2^2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^2 (2+m)^2 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{5 b c d_1^2 d_2^2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^2 (2+m) (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5 b c^3 d_1^2 d_2^2 (f x)^{4+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^4 (4+m)^2 (6+m) \sqrt{-1+c x} \sqrt{1+c x}} + \frac{2 b c^3 d_1^2 d_2^2 (f x)^{4+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^4 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{b c^5 d_1^2 d_2^2 (f x)^{6+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^6 (6+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{15 d_1^2 d_2^2 (f x)^{1+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x} (a + b \operatorname{ArcCosh}[c x])}{f (6+m) (8+6m+m^2)} + \\
& \frac{5 d_1 d_2 (f x)^{1+m} (d_1 + c d_1 x)^{3/2} (d_2 - c d_2 x)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{f (4+m) (6+m)} + \frac{(f x)^{1+m} (d_1 + c d_1 x)^{5/2} (d_2 - c d_2 x)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{f (6+m)} + \\
& \left( \frac{15 d_1^2 d_2^2 (f x)^{1+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (4+m) (6+m) (2+3m+m^2) \sqrt{1-c x} \sqrt{1+c x}} \right) / \\
& \left( \frac{15 b c d_1^2 d_2^2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}} \right) /
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int (f x)^m (d_1 + c d_1 x)^{5/2} (d_2 - c d_2 x)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

**Problem 158: Unable to integrate problem.**

$$\int (f x)^m (d_1 + c d_1 x)^{3/2} (d_2 - c d_2 x)^{3/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 503 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 b c d_1 d_2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^2 (2+m)^2 (4+m) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c d_1 d_2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^2 (2+m) (4+m) \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c^3 d_1 d_2 (f x)^{4+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}{f^4 (4+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{3 d_1 d_2 (f x)^{1+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x} (a + b \operatorname{ArcCosh}[c x])}{f (8+6m+m^2)} + \frac{(f x)^{1+m} (d_1 + c d_1 x)^{3/2} (d_2 - c d_2 x)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{f (4+m)} + \\
& \frac{3 d_1 d_2 (f x)^{1+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (4+m) (2+3m+m^2) \sqrt{1-c x} \sqrt{1+c x}} - \\
& \left( \frac{3 b c d_1 d_2 (f x)^{2+m} \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 (4+m) \sqrt{-1+c x} \sqrt{1+c x}} \right) /
\end{aligned}$$



Result (type 8, 37 leaves):

$$\int (f x)^m (d1 + c d1 x)^{3/2} (d2 - c d2 x)^{3/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Problem 159: Unable to integrate problem.

$$\int (f x)^m \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 302 leaves, 3 steps):

$$\begin{aligned} & - \frac{b c (f x)^{2+m} \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}}{f^2 (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{(f x)^{1+m} \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x} (a + b \operatorname{ArcCosh}[c x])}{f (2+m)} + \\ & \frac{(f x)^{1+m} \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (2+3m+m^2) \sqrt{1 - c x} \sqrt{1 + c x}} - \\ & \frac{b c (f x)^{2+m} \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 8, 37 leaves):

$$\int (f x)^m \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x} (a + b \operatorname{ArcCosh}[c x]) dx$$

Problem 160: Unable to integrate problem.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{\sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} dx$$

Optimal (type 5, 188 leaves, 1 step):

$$\begin{aligned} & \frac{(f x)^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (1+m) \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} + \\ & \frac{b c (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m) \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} \end{aligned}$$

Result (type 9, 322 leaves):

$$\begin{aligned}
& - \frac{1}{8 c d_1 \sqrt{d_2 - c d_2 x}} (f x)^m \sqrt{d_1 + c d_1 x} \\
& \left( - \left( \left( 8 a (-1 + m) (1 + c x) \operatorname{AppellF1} \left[ -m, -m, \frac{1}{2}, 1 - m, \frac{1}{1 + c x}, \frac{2}{1 + c x} \right] \right) / \left( m \left( m \operatorname{AppellF1} \left[ 1 - m, 1 - m, \frac{1}{2}, 2 - m, \frac{1}{1 + c x}, \frac{2}{1 + c x} \right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[ 1 - m, -m, \frac{3}{2}, 2 - m, \frac{1}{1 + c x}, \frac{2}{1 + c x} \right] + (-1 + m) (1 + c x) \operatorname{AppellF1} \left[ -m, -m, \frac{1}{2}, 1 - m, \frac{1}{1 + c x}, \frac{2}{1 + c x} \right] \right) \right) \right) + \\
& b \left( \frac{4 \sqrt{\frac{-1 + c x}{1 + c x}} \operatorname{ArcCosh}[c x] \operatorname{Hypergeometric2F1} \left[ 1, \frac{2 + m}{2}, \frac{3 + m}{2}, c^2 x^2 \right]}{1 + m} - \right. \\
& \quad \left. \frac{2^{-m} c \sqrt{\pi} x \operatorname{Gamma}[1 + m] \operatorname{HypergeometricPFQRegularized} \left[ \left\{ 1, \frac{2 + m}{2}, \frac{2 + m}{2} \right\}, \left\{ \frac{3 + m}{2}, \frac{4 + m}{2} \right\}, c^2 x^2 \right]}{1 + c x} \operatorname{Sinh}[2 \operatorname{ArcCosh}[c x]] \right)
\end{aligned}$$

### Problem 161: Unable to integrate problem.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{(d_1 + c d_1 x)^{3/2} (d_2 - c d_2 x)^{3/2}} dx$$

Optimal (type 5, 336 leaves, 4 steps):

$$\begin{aligned}
& \frac{(f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{d_1 d_2 f \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}} - \frac{m (f x)^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2 \right]}{d_1 d_2 f (1 + m) \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}} + \\
& \frac{b c (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Hypergeometric2F1} \left[ 1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2 \right]}{d_1 d_2 f^2 (2 + m) \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}} - \\
& \frac{b c m (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{HypergeometricPFQ} \left[ \left\{ 1, 1 + \frac{m}{2}, 1 + \frac{m}{2} \right\}, \left\{ \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2} \right\}, c^2 x^2 \right]}{d_1 d_2 f^2 (1 + m) (2 + m) \sqrt{d_1 + c d_1 x} \sqrt{d_2 - c d_2 x}}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{(d1 + c d1 x)^{3/2} (d2 - c d2 x)^{3/2}} dx$$

**Problem 162: Unable to integrate problem.**

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{(d1 + c d1 x)^{5/2} (d2 - c d2 x)^{5/2}} dx$$

Optimal (type 5, 504 leaves, 7 steps):

$$\begin{aligned} & \frac{(f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{3 d1 d2 f (d1 + c d1 x)^{3/2} (d2 - c d2 x)^{3/2}} + \frac{(2 - m) (f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{3 d1^2 d2^2 f \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} - \\ & \frac{(2 - m) m (f x)^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{3 d1^2 d2^2 f (1+m) \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} + \\ & \frac{b c (2 - m) (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d1^2 d2^2 f^2 (2+m) \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} + \\ & \frac{b c (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d1^2 d2^2 f^2 (2+m) \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} - \\ & \frac{b c (2 - m) m (f x)^{2+m} \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{3 d1^2 d2^2 f^2 (1+m) (2+m) \sqrt{d1 + c d1 x} \sqrt{d2 - c d2 x}} \end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])}{(d1 + c d1 x)^{5/2} (d2 - c d2 x)^{5/2}} dx$$

**Problem 163: Unable to integrate problem.**

$$\int \frac{(f x)^m \operatorname{ArcCosh}[a x]}{\sqrt{1 - a^2 x^2}} dx$$

Optimal (type 5, 128 leaves, 1 step):

$$\frac{(f x)^{1+m} \operatorname{ArcCosh}[a x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{f (1+m)} + \frac{a (f x)^{2+m} \sqrt{-1 + a x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, a^2 x^2\right]}{f^2 (1+m) (2+m) \sqrt{1 - a x}}$$

Result (type 9, 164 leaves):

$$-\frac{1}{2\sqrt{-(-1+ax)(1+ax)}}x(fx)^m\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(\frac{2\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\operatorname{ArcCosh}[ax]\operatorname{Hypergeometric2F1}\left[1, 1+\frac{m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m}-\right. \\ \left.2^{-1-m}a\sqrt{\pi}x\operatorname{Gamma}[1+m]\operatorname{HypergeometricPFQRegularized}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3+m}{2}, 2+\frac{m}{2}\right\}, a^2x^2\right]\right)$$

Problem 176: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{ArcCosh}[cx])^2}{x^3} dx$$

Optimal (type 4, 427 leaves, 12 steps):

$$-\frac{bc\sqrt{d-c^2dx^2}(a+b\operatorname{ArcCosh}[cx])}{x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{ArcCosh}[cx])^2}{2x^2}+ \\ \frac{c^2\sqrt{d-c^2dx^2}(a+b\operatorname{ArcCosh}[cx])^2\operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b^2c^2\sqrt{d-c^2dx^2}\operatorname{ArcTan}\left[\sqrt{-1+cx}\sqrt{1+cx}\right]}{\sqrt{-1+cx}\sqrt{1+cx}}- \\ \frac{ibc^2\sqrt{d-c^2dx^2}(a+b\operatorname{ArcCosh}[cx])\operatorname{PolyLog}\left[2, -ie^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{ibc^2\sqrt{d-c^2dx^2}(a+b\operatorname{ArcCosh}[cx])\operatorname{PolyLog}\left[2, ie^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{-1+cx}\sqrt{1+cx}}+ \\ \frac{ib^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left[3, -ie^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{ib^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left[3, ie^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Result (type 4, 5160 leaves):

$$-\frac{a^2\sqrt{-d(-1+c^2x^2)}}{2x^2}-\frac{1}{2}a^2c^2\sqrt{d}\operatorname{Log}[x]+\frac{1}{2}a^2c^2\sqrt{d}\operatorname{Log}\left[d+\sqrt{d}\sqrt{-d(-1+c^2x^2)}\right]+ \\ \frac{1}{\sqrt{-d(-1+cx)(1+cx)}}iab^2c^2d\left(-\frac{ie^{\sqrt{\frac{-1+cx}{1+cx}}}(1+cx)}{cx}-\frac{i(-1+cx)(1+cx)\operatorname{ArcCosh}[cx]}{c^2x^2}\right)+$$

$$\begin{aligned}
& \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[cx]}\right] - \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[cx]}\right] + \\
& \left. \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[cx]}\right] - \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[cx]}\right] \right) + b^2 c^2 \\
& \left( \frac{d \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx] \left(2 + \frac{\sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx]}{cx}\right)}{2cx \sqrt{-d(-1+cx)(1+cx)}} - \frac{1}{2} d \frac{2 \sqrt{\frac{-1+cx}{1+cx}} (1+cx) (2 + \operatorname{ArcCosh}[cx]^2) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right]}{\sqrt{-d(-1+cx)(1+cx)}} - \right. \\
& \left. \left( 2 \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx] \left( 2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \left( \operatorname{Log}\left[1-i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] \right) - \operatorname{Log}\left[1+i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] \right) \right) \right) \right) / \\
& \left( \sqrt{-d(-1+cx)(1+cx)} \right) + 2 \left( \frac{1}{2 \sqrt{-d(-1+cx)(1+cx)}} \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \left( -2 \operatorname{ArcCosh}[cx]^2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] - \right. \right. \\
& \left. \left. i \operatorname{ArcCosh}[cx]^2 \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[cx]}\right] + i \operatorname{ArcCosh}[cx]^2 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[cx]}\right] - \right. \right. \\
& \left. \left. 4 \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[1 - \frac{i(1+cx)(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])^2}{2cx}\right] - \right. \right. \\
& \left. \left. 2i \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[cx]}\right] \operatorname{Log}\left[1 - \frac{i(1+cx)(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])^2}{2cx}\right] + \right. \right. \\
& \left. \left. 2i \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[cx]}\right] \operatorname{Log}\left[1 - \frac{i(1+cx)(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])^2}{2cx}\right] + \right. \right. \\
& \left. \left. 4 \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[1 + \frac{i(1+cx)(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])^2}{2cx}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 i \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1+\frac{i(1+c x)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right]- \\
& 2 i \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1+\frac{i(1+c x)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right]- \\
& 2 i\left(\operatorname{Log}\left[1-\frac{i(1+c x)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right]-\operatorname{Log}\left[1+\frac{i(1+c x)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right]\right) \operatorname{PolyLog}[2, \\
& -i e^{-\operatorname{ArcCosh}[c x]}]+2 i\left(\operatorname{Log}\left[1-\frac{i(1+c x)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right]-\operatorname{Log}\left[1+\frac{i(1+c x)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right]\right) \\
& \operatorname{PolyLog}[2, i e^{-\operatorname{ArcCosh}[c x]}]+2 i \operatorname{PolyLog}[3,-i e^{-\operatorname{ArcCosh}[c x]}]-2 i \operatorname{PolyLog}[3, i e^{-\operatorname{ArcCosh}[c x]}] \\
& \frac{1}{\sqrt{-d(-1+c x)(1+c x)}} i \sqrt{\frac{-1+c x}{1+c x}}(1+c x)\left(\operatorname{Log}\left[i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right]^2 \operatorname{Log}\left[\frac{1}{1-\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right]-\right. \\
& \left.\operatorname{Log}\left[-i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right]^2 \operatorname{Log}\left[-\frac{2}{-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right]-2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1-\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]+2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]^2- \\
& \left.\operatorname{Log}\left[-i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right]^2 \operatorname{Log}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right]+\operatorname{Log}\left[1-\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \right. \\
& \left.\operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right]-\operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]^2 \operatorname{Log}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right]+\operatorname{Log}\left[-i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right]^2 \operatorname{Log}\left[\frac{(1-i)\left(-i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right]+ \\
& 2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1-\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[\frac{(1-i)\left(-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right]- \\
& 2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[\frac{(1-i)\left(-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right]+
\end{aligned}$$











$$\left. \left. \left. \left. \left. 2 \operatorname{PolyLog}\left[3, -i \left( c x + \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \right) \right] + 2 \operatorname{PolyLog}\left[3, i \left( c x + \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \right) \right] \right) \right) \right) \right) \right)$$

**Problem 184: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{x^3} dx$$

Optimal (type 4, 630 leaves, 18 steps):

$$\begin{aligned} & -2 b^2 c^2 d \sqrt{d - c^2 d x^2} + \frac{3 a b c^3 d x \sqrt{d - c^2 d x^2}}{\sqrt{-1+cx} \sqrt{1+cx}} + \frac{3 b^2 c^3 d x \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{\sqrt{-1+cx} \sqrt{1+cx}} - \frac{b c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{x \sqrt{-1+cx} \sqrt{1+cx}} - \\ & \frac{b c^3 d x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{-1+cx} \sqrt{1+cx}} - \frac{3}{2} c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{2 x^2} + \\ & \frac{3 c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1+cx} \sqrt{1+cx}} + \frac{b^2 c^2 d \sqrt{d - c^2 d x^2} \operatorname{ArcTan}\left[\sqrt{-1+cx} \sqrt{1+cx}\right]}{\sqrt{-1+cx} \sqrt{1+cx}} - \\ & \frac{3 i b c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1+cx} \sqrt{1+cx}} + \frac{3 i b c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{3 i b^2 c^2 d \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1+cx} \sqrt{1+cx}} - \frac{3 i b^2 c^2 d \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

Result (type 4, 5484 leaves):

$$\begin{aligned} & \left( -a^2 c^2 d - \frac{a^2 d}{2 x^2} \right) \sqrt{-d (-1 + c^2 x^2)} - \frac{3}{2} a^2 c^2 d^{3/2} \operatorname{Log}[x] + \\ & \frac{3}{2} a^2 c^2 d^{3/2} \operatorname{Log}\left[ d + \sqrt{d} \sqrt{-d (-1 + c^2 x^2)} \right] - 2 a b c^2 d \sqrt{-d (-1 + c x) (1 + c x)} \left( - \frac{c x}{\sqrt{\frac{-1+cx}{1+cx}} (1+cx)} + \operatorname{ArcCosh}[c x] + \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{i \operatorname{ArcCosh}[c x] \left( \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c x]}\right] - \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c x]}\right] \right) + \frac{i \left( \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c x]}\right] - \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c x]}\right] \right)}{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \right) + \\
& \frac{1}{\sqrt{-d(-1+c x)(1+c x)}} i a b c^2 d^2 \left( -\frac{i \sqrt{\frac{-1+c x}{1+c x}} (1+c x)}{c x} - \frac{i(-1+c x)(1+c x) \operatorname{ArcCosh}[c x]}{c^2 x^2} + \right. \\
& \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c x]}\right] - \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& \left. \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c x]}\right] - \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c x]}\right] \right) - \\
& b^2 c^2 d \sqrt{-d(-1+c x)(1+c x)} \left( 2 - \frac{2 c x \operatorname{ArcCosh}[c x]}{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)} + \operatorname{ArcCosh}[c x]^2 + \frac{1}{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \right. \\
& \left. i \left( \operatorname{ArcCosh}[c x]^2 \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c x]}\right] - \operatorname{ArcCosh}[c x]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c x]}\right] + 2 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c x]}\right] - \right. \right. \\
& \left. \left. 2 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c x]}\right] + 2 \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c x]}\right] - 2 \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcCosh}[c x]}\right] \right) \right) + \\
& b^2 c^2 d \left( \frac{d \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \left( 2 + \frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{c x} \right)}{2 c x \sqrt{-d(-1+c x)(1+c x)}} + \frac{1}{2 \sqrt{-d(-1+c x)(1+c x)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{i d} \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \left( 4 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] + \text{ArcCosh} [c x]^2 \text{Log} \left[ 1 - \text{i} e^{-\text{ArcCosh} [c x]} \right] - \text{ArcCosh} [c x]^2 \text{Log} \left[ 1 + \text{i} e^{-\text{ArcCosh} [c x]} \right] - \right. \\
& 4 \text{i ArcCosh} [c x] \text{ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ 1 - \text{i} e^{2 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right]} \right] + 4 \text{i ArcCosh} [c x] \text{ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \left. \right] \\
& \text{Log} \left[ 1 + \text{i} e^{2 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right]} \right] + 2 \text{Log} \left[ \text{i} \left( c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \right) \right]^2 \text{Log} \left[ \frac{1}{1 - \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right]} \right] - \\
& 2 \text{Log} \left[ -\text{i} \left( c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \right) \right]^2 \text{Log} \left[ -\frac{2}{-1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right]} \right] - 4 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ \right. \\
& \left. 1 - \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] + 4 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right]^2 - \\
& 2 \text{Log} \left[ -\text{i} \left( c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \right) \right]^2 \text{Log} \left[ \left( \frac{1}{2} + \frac{\text{i}}{2} \right) \left( -\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right) \right] + 2 \text{Log} \left[ 1 - \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \\
& \text{Log} \left[ -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ \left( \frac{1}{2} + \frac{\text{i}}{2} \right) \left( -\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right) \right] - 2 \text{Log} \left[ -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right]^2 \\
& \text{Log} \left[ \left( \frac{1}{2} + \frac{\text{i}}{2} \right) \left( -\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right) \right] + 2 \text{Log} \left[ -\text{i} \left( c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \right) \right]^2 \text{Log} \left[ \frac{(1 - \text{i}) \left( -\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right)}{-1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right]} \right] + \\
& 4 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ 1 - \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ \frac{(1 - \text{i}) \left( -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right)}{\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right]} \right] - \\
& 4 \text{i ArcTan} \left[ \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ \frac{(1 - \text{i}) \left( -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right)}{\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right]} \right] + \\
& 2 \text{Log} \left[ -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right] \text{Log} \left[ \left( \frac{1}{2} + \frac{\text{i}}{2} \right) \left( -\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right) \right] \text{Log} \left[ \frac{(1 - \text{i}) \left( -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right)}{\text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right]} \right] + \\
& 2 \text{Log} \left[ \text{i} \left( c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \right) \right]^2 \text{Log} \left[ (1 - \text{i}) \left( \text{i} + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh} [c x] \right] \right) \right] -
\end{aligned}$$











$$\begin{aligned}
& 2 \operatorname{Log} \left[ \frac{(1+i) \left(1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right)}{i + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right]} \right] \operatorname{PolyLog} \left[ 2, \left( \frac{1-i}{2} - \frac{i}{2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right) \right] + \\
& 4 \operatorname{Log} \left[ i \left( c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \right) \right] \operatorname{PolyLog} \left[ 2, \left( \frac{1}{2} + \frac{i}{2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right) \right] + \\
& 2 \operatorname{Log} \left[ 1 - \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right] \operatorname{PolyLog} \left[ 2, \left( \frac{1}{2} + \frac{i}{2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right) \right] + 2 \operatorname{Log} \left[ \frac{(1-i) \left( -1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right)}{i + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right]} \right] \\
& \operatorname{PolyLog} \left[ 2, \left( \frac{1}{2} + \frac{i}{2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right) \right] - 2 \operatorname{Log} \left[ 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right] \operatorname{PolyLog} \left[ 2, \left( \frac{1}{2} + \frac{i}{2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right) \right] - \\
& 2 \operatorname{Log} \left[ \frac{(1+i) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right)}{i + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right]} \right] \operatorname{PolyLog} \left[ 2, \left( \frac{1}{2} + \frac{i}{2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[c x] \right] \right) \right] - 2 \operatorname{PolyLog} \left[ 3, -i e^{-\operatorname{ArcCosh}[c x]} \right] + \\
& \left. 2 \operatorname{PolyLog} \left[ 3, i e^{-\operatorname{ArcCosh}[c x]} \right] - 4 \operatorname{PolyLog} \left[ 3, -i \left( c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \right) \right] + 4 \operatorname{PolyLog} \left[ 3, i \left( c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \right) \right] \right]
\end{aligned}$$

**Problem 192: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{x^3} dx$$

Optimal (type 4, 890 leaves, 28 steps):

$$\begin{aligned}
& -\frac{170}{27} b^2 c^2 d^2 \sqrt{d - c^2 d x^2} + \frac{5}{27} b^2 c^4 d^2 x^2 \sqrt{d - c^2 d x^2} + \frac{5 a b c^3 d^2 x \sqrt{d - c^2 d x^2}}{\sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{5 b^2 c^2 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2}}{3 (1 - c x) (1 + c x)} + \frac{b^2 c^2 d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2}}{9 (1 - c x) (1 + c x)} + \frac{5 b^2 c^3 d^2 x \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{x \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^3 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 b c^5 d^2 x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{9 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{5}{2} c^2 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 - \frac{5}{6} c^2 d (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2 - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{2 x^2} + \\
& \frac{5 c^2 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 \operatorname{ArcTan}[e^{\operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b^2 c^2 d^2 \sqrt{-1 + c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{ArcTan}[\sqrt{-1 + c^2 x^2}]}{(1 - c x) (1 + c x)} - \\
& \frac{5 i b c^2 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}[2, -i e^{\operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 i b c^2 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}[2, i e^{\operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{5 i b^2 c^2 d^2 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[3, -i e^{\operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{5 i b^2 c^2 d^2 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[3, i e^{\operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 5734 leaves):

$$\begin{aligned}
& \sqrt{-d (-1 + c^2 x^2)} \left( -\frac{7}{3} a^2 c^2 d^2 - \frac{a^2 d^2}{2 x^2} + \frac{1}{3} a^2 c^4 d^2 x^2 \right) - \frac{1}{18 \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)} \\
& a b c^2 d^2 \sqrt{-d (-1 + c x) (1 + c x)} \left( -9 c x - 12 \left( \frac{-1 + c x}{1 + c x} \right)^{3/2} (1 + c x)^3 \operatorname{ArcCosh}[c x] + \operatorname{Cosh}[3 \operatorname{ArcCosh}[c x]] \right) + \frac{1}{54} b^2 c^2 d^2 \sqrt{-d (-1 + c x) (1 + c x)} \\
& \left( -26 + \frac{27 c x \operatorname{ArcCosh}[c x]}{\sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)} - 9 \operatorname{ArcCosh}[c x]^2 + (2 + 9 \operatorname{ArcCosh}[c x]^2) \operatorname{Cosh}[2 \operatorname{ArcCosh}[c x]] - \frac{3 \operatorname{ArcCosh}[c x] \operatorname{Cosh}[3 \operatorname{ArcCosh}[c x]]}{\sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)} \right) - \\
& \frac{5}{2} a^2 c^2 d^{5/2} \operatorname{Log}[x] + \frac{5}{2} a^2 c^2 d^{5/2} \operatorname{Log}[d + \sqrt{d} \sqrt{-d (-1 + c^2 x^2)}] - \\
& 4 a b c^2 d^2 \sqrt{-d (-1 + c x) (1 + c x)} \left( -\frac{c x}{\sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)} + \operatorname{ArcCosh}[c x] + \frac{i \operatorname{ArcCosh}[c x] (\operatorname{Log}[1 - i e^{-\operatorname{ArcCosh}[c x]}] - \operatorname{Log}[1 + i e^{-\operatorname{ArcCosh}[c x]}])}{\sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)} \right) +
\end{aligned}$$

$$\left. \frac{i \left( \text{PolyLog}\left[2, -i e^{-\text{ArcCosh}[c x]}\right] - \text{PolyLog}\left[2, i e^{-\text{ArcCosh}[c x]}\right] \right)}{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \right\} + \frac{1}{\sqrt{-d (-1+c x) (1+c x)}} i a b c^2 d^3$$

$$\left( -\frac{i \sqrt{\frac{-1+c x}{1+c x}} (1+c x)}{c x} - \frac{i (-1+c x) (1+c x) \text{ArcCosh}[c x]}{c^2 x^2} + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{ArcCosh}[c x] \text{Log}\left[1 - i e^{-\text{ArcCosh}[c x]}\right] - \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \right.$$

$$\left. \text{ArcCosh}[c x] \text{Log}\left[1 + i e^{-\text{ArcCosh}[c x]}\right] + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{PolyLog}\left[2, -i e^{-\text{ArcCosh}[c x]}\right] - \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{PolyLog}\left[2, i e^{-\text{ArcCosh}[c x]}\right] \right) -$$

$$2 b^2 c^2 d^2 \sqrt{-d (-1+c x) (1+c x)} \left( 2 - \frac{2 c x \text{ArcCosh}[c x]}{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)} + \text{ArcCosh}[c x]^2 + \frac{1}{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \right.$$

$$\left. i \left( \text{ArcCosh}[c x]^2 \text{Log}\left[1 - i e^{-\text{ArcCosh}[c x]}\right] - \text{ArcCosh}[c x]^2 \text{Log}\left[1 + i e^{-\text{ArcCosh}[c x]}\right] + 2 \text{ArcCosh}[c x] \text{PolyLog}\left[2, -i e^{-\text{ArcCosh}[c x]}\right] - \right. \right.$$

$$\left. \left. 2 \text{ArcCosh}[c x] \text{PolyLog}\left[2, i e^{-\text{ArcCosh}[c x]}\right] + 2 \text{PolyLog}\left[3, -i e^{-\text{ArcCosh}[c x]}\right] - 2 \text{PolyLog}\left[3, i e^{-\text{ArcCosh}[c x]}\right] \right) \right) +$$

$$b^2 c^2 d^2 \left( \frac{d \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{ArcCosh}[c x] \left( 2 + \frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{ArcCosh}[c x]}{c x} \right)}{2 c x \sqrt{-d (-1+c x) (1+c x)}} + \frac{1}{2 \sqrt{-d (-1+c x) (1+c x)}} \right)$$

$$i d \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \left( 4 i \text{ArcTan}\left[\text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]\right] + \text{ArcCosh}[c x]^2 \text{Log}\left[1 - i e^{-\text{ArcCosh}[c x]}\right] - \text{ArcCosh}[c x]^2 \text{Log}\left[1 + i e^{-\text{ArcCosh}[c x]}\right] - \right.$$













$$\begin{aligned}
& 2 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] + 2 \operatorname{Log}\left[\frac{(1-i) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \\
& \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - 2 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - \\
& 2 \operatorname{Log}\left[\frac{(1+i) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - 2 \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& \left. 2 \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcCosh}[c x]}\right] - 4 \operatorname{PolyLog}\left[3, -i \left(c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)\right] + 4 \operatorname{PolyLog}\left[3, i \left(c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)\right]\right]
\end{aligned}$$

**Problem 199: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x])^3}{3 b c \sqrt{d - c^2 d x^2}}$$

Result (type 3, 147 leaves):

$$\frac{3 a b \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]^2}{\sqrt{d - c^2 d x^2}} + \frac{b^2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]^3}{\sqrt{d - c^2 d x^2}} - \frac{3 a^2 \operatorname{ArcTan}\left[\frac{c x \sqrt{d - c^2 d x^2}}{\sqrt{d} (-1+c^2 x^2)}\right]}{\sqrt{d}}$$

3 c

**Problem 202: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{x^3 \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 4, 430 leaves, 12 steps):

$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{x \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{2 d x^2} + \frac{c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} \\
& \frac{b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcTan}\left[\sqrt{-1+c x} \sqrt{1+c x}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 5076 leaves):

$$\begin{aligned}
& -\frac{a^2 \sqrt{-d(-1+c^2 x^2)}}{2 d x^2} + \frac{a^2 c^2 \operatorname{Log}[x]}{2 \sqrt{d}} - \frac{a^2 c^2 \operatorname{Log}\left[d+\sqrt{d} \sqrt{-d(-1+c^2 x^2)}\right]}{2 \sqrt{d}} + \frac{1}{\sqrt{-d(-1+c x)(1+c x)}} \\
& a b c^2 \left( \frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)}{c x} + \frac{(-1+c x)(1+c x) \operatorname{ArcCosh}[c x]}{c^2 x^2} - i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c x]}\right] + \right. \\
& \left. i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c x]}\right] - i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c x]}\right] + \right. \\
& \left. i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c x]}\right] \right) + b^2 c^2 \left( \frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \left(2 + \frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{c x}\right)}{2 c x \sqrt{-d(-1+c x)(1+c x)}} - \right. \\
& \left. \frac{1}{2 \sqrt{-d(-1+c x)(1+c x)}} i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \left( -4 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] + \operatorname{ArcCosh}[c x]^2 \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c x]}\right] - \right. \right. \\
& \left. \left. \operatorname{ArcCosh}[c x]^2 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c x]}\right] - 4 i \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1-i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] \right) + \right.
\end{aligned}$$







$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1 - \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] - \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1 - \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] + \\
& 4 i \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1 + \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] - \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1 + \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1 + \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] + \\
& 2 \left( \operatorname{Log}\left[1 - \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] - \operatorname{Log}\left[1 + \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] \right) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c x]}\right] - \\
& 2 \left( \operatorname{Log}\left[1 - \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] - \operatorname{Log}\left[1 + \frac{i(1+cx)\left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2cx}\right] \right) \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}\left[2, -i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] + 2 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, -i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] - \\
& 2 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, -i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] - 2 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}\left[2, i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] - \\
& 2 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] + 2 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \\
& \operatorname{PolyLog}\left[2, i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}\right] - 4 \operatorname{Log}\left[-i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] \operatorname{PolyLog}\left[2, -c x - \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right] + \\
& 4 \operatorname{Log}\left[i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] \operatorname{PolyLog}\left[2, -c x - \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right] + 4 \operatorname{Log}\left[-i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] \\
& \operatorname{PolyLog}\left[2, -i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] - 4 \operatorname{Log}\left[i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] \operatorname{PolyLog}\left[2, i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] + \\
& 4 \operatorname{Log}\left[-i \left(c x + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right] \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] +
\end{aligned}$$





$$\begin{aligned}
& 2 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] + 2 \operatorname{Log}\left[\frac{(1-i) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \\
& \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - 2 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - \\
& 2 \operatorname{Log}\left[\frac{(1+i) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - 2 \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& \left. 2 \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcCosh}[c x]}\right] - 4 \operatorname{PolyLog}\left[3, -i \left(c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)\right] + 4 \operatorname{PolyLog}\left[3, i \left(c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)\right]\right]
\end{aligned}$$

**Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{x^3 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 650 leaves, 27 steps):

$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x])}{d x \sqrt{d - c^2 d x^2}} + \frac{3 c^2 (a + b \operatorname{ArcCosh}[c x])^2}{2 d \sqrt{d - c^2 d x^2}} - \\
& \frac{(a + b \operatorname{ArcCosh}[c x])^2}{2 d x^2 \sqrt{d - c^2 d x^2}} + \frac{3 c^2 \sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcTan}\left[\sqrt{-1+c x} \sqrt{1+c x}\right]}{d \sqrt{d - c^2 d x^2}} + \frac{4 b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{2 b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} - \frac{3 i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{3 i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} - \frac{2 b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{3 i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} - \frac{3 i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c x]}\right]}{d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 5400 leaves):

$$\begin{aligned}
& \sqrt{-d(-1+c^2x^2)} \left( -\frac{a^2}{2d^2x^2} - \frac{a^2c^2}{d^2(-1+c^2x^2)} \right) + \frac{3a^2c^2 \operatorname{Log}[x]}{2d^{3/2}} - \frac{3a^2c^2 \operatorname{Log}[d + \sqrt{d} \sqrt{-d(-1+c^2x^2)}]}{2d^{3/2}} - \\
& \frac{1}{d} b^2 c^2 \left( \frac{1}{2\sqrt{-d(-1+cx)}(1+cx)} - i \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \left( -4i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] + 3 \operatorname{ArcCosh}[cx]^2 \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[cx]}\right] - \right. \right. \\
& \left. \left. 3 \operatorname{ArcCosh}[cx]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[cx]}\right] - 12i \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[1 - i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right]}\right] \right) + \right. \\
& \left. 12i \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[1 + i e^{2i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right]}\right] + 6 \operatorname{Log}\left[i \left( cx + \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \right)^2 \right. \right. \\
& \left. \left. \operatorname{Log}\left[\frac{1}{1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] - 6 \operatorname{Log}\left[-i \left( cx + \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \right)^2 \operatorname{Log}\left[-\frac{2}{-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] - \right. \right. \\
& \left. \left. 12i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] + \right. \right. \\
& \left. \left. 12i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right]^2 - 6 \operatorname{Log}\left[-i \left( cx + \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \right)^2 \right. \right. \\
& \left. \left. \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right)\right] + 6 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \right. \right. \\
& \left. \left. \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right)\right] - 6 \operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right]^2 \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right)\right] \right) + \\
& 6 \operatorname{Log}\left[-i \left( cx + \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \right)^2 \operatorname{Log}\left[\frac{(1-i) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right)}{-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] + \right. \\
& \left. 12i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[\frac{(1-i) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] - \right. \\
& \left. 12i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right] \operatorname{Log}\left[\frac{(1-i) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}\right] \right) +
\end{aligned}$$









$$\begin{aligned}
& 6 \operatorname{Log}\left[\frac{(1+i)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]- \\
& 12 \operatorname{Log}\left[-i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]- \\
& 6 \operatorname{Log}\left[1-\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]- \\
& 6 \operatorname{Log}\left[\frac{(1-i)\left(-1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]+ \\
& 6 \operatorname{Log}\left[1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]+ \\
& 6 \operatorname{Log}\left[\frac{(1+i)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]+ \\
& 12 \operatorname{Log}\left[i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]+ \\
& 6 \operatorname{Log}\left[1-\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]+ \\
& 6 \operatorname{Log}\left[\frac{(1-i)\left(-1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]- \\
& 6 \operatorname{Log}\left[1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]- \\
& 6 \operatorname{Log}\left[\frac{(1+i)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)}{i+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[c x]\right]\right)\right]-6 \operatorname{PolyLog}\left[3,-i e^{-\operatorname{ArcCosh}[c x]}\right]+ \\
& 6 \operatorname{PolyLog}\left[3,i e^{-\operatorname{ArcCosh}[c x]}\right]-12 \operatorname{PolyLog}\left[3,-i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right]+12 \operatorname{PolyLog}\left[3,i\left(c x+\sqrt{\frac{-1+c x}{1+c x}}(1+c x)\right)\right]+ \\
& \frac{1}{2 \sqrt{-d(-1+c x)(1+c x)}}\left(4 \sqrt{\frac{-1+c x}{1+c x}}(1+c x) \operatorname{PolyLog}\left[2,-e^{-\operatorname{ArcCosh}[c x]}\right]-4 \sqrt{\frac{-1+c x}{1+c x}}(1+c x) \operatorname{PolyLog}\left[2,e^{-\operatorname{ArcCosh}[c x]}\right]-\right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcCosh}[c x] \left( \frac{2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x)}{c x} + \frac{(-1+c x) (1+c x) \text{ArcCosh}[c x]}{c^2 x^2} + 2 \text{ArcCosh}[c x] \text{Cosh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]^2 - \right. \\
& \left. 4 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{Log}\left[1 - e^{-\text{ArcCosh}[c x]}\right] + 4 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{Log}\left[1 + e^{-\text{ArcCosh}[c x]}\right] - 2 \text{ArcCosh}[c x] \text{Sinh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]^2 \right) \\
& \frac{1}{d \sqrt{-d (-1+c x) (1+c x)}} a b c^2 \left( -\frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x)}{c x} - \frac{(-1+c x) (1+c x) \text{ArcCosh}[c x]}{c^2 x^2} - \right. \\
& 2 \text{ArcCosh}[c x] \text{Cosh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]^2 + \\
& 3 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{ArcCosh}[c x] \text{Log}\left[1 - i e^{-\text{ArcCosh}[c x]}\right] - \\
& 3 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{ArcCosh}[c x] \text{Log}\left[1 + i e^{-\text{ArcCosh}[c x]}\right] - \\
& 2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{Log}\left[\text{Cosh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]\right] + \\
& 2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{Log}\left[\text{Sinh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]\right] + \\
& 3 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{PolyLog}\left[2, -i e^{-\text{ArcCosh}[c x]}\right] - \\
& 3 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{PolyLog}\left[2, i e^{-\text{ArcCosh}[c x]}\right] +
\end{aligned}$$



$$\left( \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx] \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right] \right)$$

Problem 222: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[cx])^2}{x^3 (d-c^2 dx^2)^{5/2}} dx$$

Optimal (type 4, 796 leaves, 41 steps):

$$\begin{aligned} & -\frac{b^2 c^2}{3 d^2 \sqrt{d-c^2 dx^2}} + \frac{bc \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}{d^2 x (1-c^2 x^2) \sqrt{d-c^2 dx^2}} - \frac{2 b c^3 x \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}{3 d^2 (1-c^2 x^2) \sqrt{d-c^2 dx^2}} + \frac{5 c^2 (a+b \operatorname{ArcCosh}[cx])^2}{6 d (d-c^2 dx^2)^{3/2}} \\ & \frac{(a+b \operatorname{ArcCosh}[cx])^2}{2 d x^2 (d-c^2 dx^2)^{3/2}} + \frac{5 c^2 (a+b \operatorname{ArcCosh}[cx])^2}{2 d^2 \sqrt{d-c^2 dx^2}} + \frac{5 c^2 \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[cx]}\right]}{d^2 \sqrt{d-c^2 dx^2}} \\ & \frac{b^2 c^2 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{ArcTan}\left[\sqrt{-1+cx} \sqrt{1+cx}\right]}{d^2 \sqrt{d-c^2 dx^2}} + \frac{26 b c^2 \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcCosh}[cx]}\right]}{3 d^2 \sqrt{d-c^2 dx^2}} + \\ & \frac{13 b^2 c^2 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCosh}[cx]}\right]}{3 d^2 \sqrt{d-c^2 dx^2}} - \frac{5 i b c^2 \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[cx]}\right]}{d^2 \sqrt{d-c^2 dx^2}} + \\ & \frac{5 i b c^2 \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[cx]}\right]}{d^2 \sqrt{d-c^2 dx^2}} - \frac{13 b^2 c^2 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCosh}[cx]}\right]}{3 d^2 \sqrt{d-c^2 dx^2}} + \\ & \frac{5 i b^2 c^2 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[cx]}\right]}{d^2 \sqrt{d-c^2 dx^2}} - \frac{5 i b^2 c^2 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[cx]}\right]}{d^2 \sqrt{d-c^2 dx^2}} \end{aligned}$$

Result (type 4, 5568 leaves):

$$\begin{aligned} & \sqrt{-d(-1+c^2 x^2)} \left( -\frac{a^2}{2 d^3 x^2} + \frac{a^2 c^2}{3 d^3 (-1+c^2 x^2)^2} - \frac{2 a^2 c^2}{d^3 (-1+c^2 x^2)} \right) + \\ & \frac{5 a^2 c^2 \operatorname{Log}[x]}{2 d^{5/2}} - \frac{5 a^2 c^2 \operatorname{Log}\left[d + \sqrt{d} \sqrt{-d(-1+c^2 x^2)}\right]}{2 d^{5/2}} + \frac{1}{6 d^2 \sqrt{-d(-1+cx)} (1+cx)} \end{aligned}$$

$$\begin{aligned}
& a b c^2 \left( \frac{6 \sqrt{\frac{-1+c x}{1+c x}} (1+c x)}{c x} + \frac{6 (-1+c x) (1+c x) \operatorname{ArcCosh}[c x]}{c^2 x^2} + 26 \operatorname{ArcCosh}[c x] \operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right] - \right. \\
& \operatorname{ArcCosh}[c x] \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - 30 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& 30 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c x]}\right] + 26 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] - \\
& 26 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] - 30 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c x]}\right] + 30 i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \\
& \left. \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c x]}\right] - 26 \operatorname{ArcCosh}[c x] \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right] - \operatorname{ArcCosh}[c x] \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 \right) + \\
& \frac{1}{d^2} b^2 c^2 \left( -\frac{1}{2 \sqrt{-d} (-1+c x) (1+c x)} i \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \left( -4 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] + 5 \operatorname{ArcCosh}[c x]^2 \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c x]}\right] - \right. \right. \\
& 5 \operatorname{ArcCosh}[c x]^2 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c x]}\right] - 20 i \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1-i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \right) + \\
& 20 i \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1+i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \right) + 10 \operatorname{Log}\left[i \left( c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \right) \right]^2 \\
& \operatorname{Log}\left[\frac{1}{1-\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] - 10 \operatorname{Log}\left[-i \left( c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \right) \right]^2 \operatorname{Log}\left[-\frac{2}{-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] - \\
& 20 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[1-\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \right) +
\end{aligned}$$







$$\begin{aligned}
& 10 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1 + \frac{i(1 + c x) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right] + \\
& 10 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c x]}\right] \operatorname{Log}\left[1 + \frac{i(1 + c x) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right] + \\
& 10 \left( \operatorname{Log}\left[1 - \frac{i(1 + c x) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right] - \operatorname{Log}\left[1 + \frac{i(1 + c x) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right] \right) \operatorname{PolyLog}[2, \\
& \quad -i e^{-\operatorname{ArcCosh}[c x]}] - 10 \left( \operatorname{Log}\left[1 - \frac{i(1 + c x) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right] - \operatorname{Log}\left[1 + \frac{i(1 + c x) \left(-i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)^2}{2 c x}\right] \right) \\
& \operatorname{PolyLog}[2, i e^{-\operatorname{ArcCosh}[c x]}] + 10 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}[2, -i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}] + \\
& 10 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}[2, -i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}] - 10 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \\
& \operatorname{PolyLog}[2, -i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}] - 10 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}[2, i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}] - \\
& 10 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}[2, i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}] + 10 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \\
& \operatorname{PolyLog}[2, i e^{2 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right]}] - 20 \operatorname{Log}\left[-i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)\right] \operatorname{PolyLog}[2, -c x - \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)] + \\
& 20 \operatorname{Log}\left[i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)\right] \operatorname{PolyLog}[2, -c x - \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)] + 20 \operatorname{Log}\left[-i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)\right] \\
& \operatorname{PolyLog}[2, -i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)] - 20 \operatorname{Log}\left[i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)\right] \operatorname{PolyLog}[2, i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)] + \\
& 20 \operatorname{Log}\left[-i \left(c x + \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)\right)\right] \operatorname{PolyLog}[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)] + \\
& 10 \operatorname{Log}\left[1 - \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)] + \\
& 10 \operatorname{Log}\left[\frac{(1 - i) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)] - \\
& 10 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)] -
\end{aligned}$$



$$\begin{aligned}
& 10 \operatorname{Log}\left[1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - \\
& 10 \operatorname{Log}\left[\frac{(1+i) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{i + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}\right] \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)\right] - 10 \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& 10 \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcCosh}[c x]}\right] - 20 \operatorname{PolyLog}\left[3, -i \left(c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)\right] + 20 \operatorname{PolyLog}\left[3, i \left(c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)\right] + \\
& \frac{1}{12 \sqrt{-d(-1+c x)(1+c x)}} \left( \frac{12 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{c x} + \frac{6(-1+c x)(1+c x) \operatorname{ArcCosh}[c x]^2}{c^2 x^2} - \right. \\
& 4 \operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 + 26 \operatorname{ArcCosh}[c x]^2 \operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - 2 \operatorname{ArcCosh}[c x] \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right] - \\
& \operatorname{ArcCosh}[c x]^2 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - 52 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& 52 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + e^{-\operatorname{ArcCosh}[c x]}\right] - 52 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcCosh}[c x]}\right] + \\
& 52 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcCosh}[c x]}\right] + 4 \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - 26 \operatorname{ArcCosh}[c x]^2 \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 - \\
& \left. \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right] - \operatorname{ArcCosh}[c x]^2 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]^2 \right) \right)
\end{aligned}$$

**Problem 250: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 4, 241 leaves, 7 steps):



$$\frac{x \operatorname{ArcCosh}[a x]^3}{c \sqrt{c - a^2 c x^2}} + \frac{\sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^3}{a c \sqrt{c - a^2 c x^2}} - \frac{3 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCosh}[a x]}\right]}{a c \sqrt{c - a^2 c x^2}} -$$

$$\frac{3 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[a x]}\right]}{a c \sqrt{c - a^2 c x^2}} + \frac{3 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCosh}[a x]}\right]}{2 a c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 212 leaves):

$$- \left( \left( i \pi^3 \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x) - 8 a x \operatorname{ArcCosh}[a x]^3 - 8 \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x) \operatorname{ArcCosh}[a x]^3 + \right. \right.$$

$$24 \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x) \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCosh}[a x]}\right] + 24 \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x) \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[a x]}\right] -$$

$$\left. \left. 12 \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x) \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCosh}[a x]}\right] \right) / \left( 8 a c \sqrt{-c (-1 + a x) (1 + a x)} \right)$$

**Problem 251: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 4, 413 leaves, 12 steps):

$$-\frac{x \operatorname{ArcCosh}[a x]}{c^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^2}{2 a c^2 (1 - a^2 x^2) \sqrt{c - a^2 c x^2}} + \frac{x \operatorname{ArcCosh}[a x]^3}{3 c (c - a^2 c x^2)^{3/2}} + \frac{2 x \operatorname{ArcCosh}[a x]^3}{3 c^2 \sqrt{c - a^2 c x^2}} +$$

$$\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^3}{3 a c^2 \sqrt{c - a^2 c x^2}} - \frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCosh}[a x]}\right]}{a c^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{Log}\left[1 - a^2 x^2\right]}{2 a c^2 \sqrt{c - a^2 c x^2}} -$$

$$\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[a x]}\right]}{a c^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCosh}[a x]}\right]}{a c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 258 leaves):

$$\frac{1}{12 a c^2 \sqrt{c - a^2 c x^2}} \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x)$$

$$\left( -i \pi^3 - \frac{12 a x \sqrt{\frac{-1+a x}{1+a x}} \operatorname{ArcCosh}[a x]}{-1 + a x} + \frac{6 \operatorname{ArcCosh}[a x]^2}{1 - a^2 x^2} + 8 \operatorname{ArcCosh}[a x]^3 + \frac{8 a x \sqrt{\frac{-1+a x}{1+a x}} \operatorname{ArcCosh}[a x]^3}{-1 + a x} - \frac{4 a x \left(\frac{-1+a x}{1+a x}\right)^{3/2} \operatorname{ArcCosh}[a x]^3}{(-1 + a x)^3} - \right.$$

$$\left. 24 \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCosh}[a x]}\right] + 12 \operatorname{Log}\left[\sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x)\right] - 24 \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[a x]}\right] + 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCosh}[a x]}\right] \right)$$

**Problem 252: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 4, 607 leaves, 20 steps):

$$-\frac{\sqrt{-1+a x} \sqrt{1+a x}}{20 a c^3 (1-a^2 x^2) \sqrt{c-a^2 c x^2}} - \frac{x \operatorname{ArcCosh}[a x]}{c^3 \sqrt{c-a^2 c x^2}} - \frac{x \operatorname{ArcCosh}[a x]}{10 c^3 (1-a x) (1+a x) \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{20 a c^3 (1-a^2 x^2)^2 \sqrt{c-a^2 c x^2}} +$$

$$\frac{2 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{5 a c^3 (1-a^2 x^2) \sqrt{c-a^2 c x^2}} + \frac{x \operatorname{ArcCosh}[a x]^3}{5 c (c-a^2 c x^2)^{5/2}} + \frac{4 x \operatorname{ArcCosh}[a x]^3}{15 c^2 (c-a^2 c x^2)^{3/2}} + \frac{8 x \operatorname{ArcCosh}[a x]^3}{15 c^3 \sqrt{c-a^2 c x^2}} +$$

$$\frac{8 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^3}{15 a c^3 \sqrt{c-a^2 c x^2}} - \frac{8 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCosh}[a x]}\right]}{5 a c^3 \sqrt{c-a^2 c x^2}} + \frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{Log}\left[1 - a^2 x^2\right]}{2 a c^3 \sqrt{c-a^2 c x^2}} -$$

$$\frac{8 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[a x]}\right]}{5 a c^3 \sqrt{c-a^2 c x^2}} + \frac{4 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCosh}[a x]}\right]}{5 a c^3 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 363 leaves):

$$\begin{aligned}
& - \frac{1}{60 a c^3 \sqrt{c - a^2 c x^2}} \sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x) \\
& \left( 4 i \pi^3 + \frac{3}{1 - a^2 x^2} + \frac{60 a x \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]}{-1 + a x} - \frac{6 a x \left(\frac{-1 + a x}{1 + a x}\right)^{3/2} \operatorname{ArcCosh}[a x]}{(-1 + a x)^3} - \frac{9 \operatorname{ArcCosh}[a x]^2}{(-1 + a^2 x^2)^2} + \frac{24 \operatorname{ArcCosh}[a x]^2}{-1 + a^2 x^2} - 32 \operatorname{ArcCosh}[a x]^3 - \right. \\
& \frac{32 a x \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^3}{-1 + a x} + \frac{16 a x \left(\frac{-1 + a x}{1 + a x}\right)^{3/2} \operatorname{ArcCosh}[a x]^3}{(-1 + a x)^3} - \frac{12 a x \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^3}{(-1 + a x)^3 (1 + a x)^2} + 96 \operatorname{ArcCosh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCosh}[a x]}\right] - \\
& \left. 60 \operatorname{Log}\left[\sqrt{\frac{-1 + a x}{1 + a x}} (1 + a x)\right] + 96 \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCosh}[a x]}\right] - 48 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCosh}[a x]}\right] \right)
\end{aligned}$$

Problem 260: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{x^3 \sqrt{1 - a^2 x^2}} dx$$

Optimal (type 4, 460 leaves, 18 steps):

$$\begin{aligned}
& \frac{3 a \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^2}{2 x \sqrt{1 - a x}} - \frac{\sqrt{1 - a^2 x^2} \operatorname{ArcCosh}[a x]^3}{2 x^2} - \frac{6 a^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x] \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} + \\
& \frac{a^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^3 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} + \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} - \\
& \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[a x]}\right]}{2 \sqrt{1 - a x}} - \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} + \\
& \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[a x]}\right]}{2 \sqrt{1 - a x}} + \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} - \\
& \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x] \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} - \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}} + \frac{3 i a^2 \sqrt{-1 + a x} \operatorname{PolyLog}\left[4, i e^{\operatorname{ArcCosh}[a x]}\right]}{\sqrt{1 - a x}}
\end{aligned}$$

Result (type 4, 1216 leaves):

$$\begin{aligned}
& \frac{1}{128 x^2 \sqrt{1 - a^2 x^2}} \\
& (1 + a x) \left( -7 i a^2 \pi^4 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} + 8 a^2 \pi^3 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x] + 192 a x \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^2 - 24 i a^2 \pi^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^2 - \right. \\
& 64 \operatorname{ArcCosh}[a x]^3 + 64 a x \operatorname{ArcCosh}[a x]^3 - 32 a^2 \pi x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^3 + \\
& 16 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^4 + 384 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x] \operatorname{Log}[1 - i e^{-\operatorname{ArcCosh}[a x]}] + \\
& 8 a^2 \pi^3 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{Log}[1 + i e^{-\operatorname{ArcCosh}[a x]}] - 384 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x] \operatorname{Log}[1 + i e^{-\operatorname{ArcCosh}[a x]}] - \\
& 48 i a^2 \pi^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x] \operatorname{Log}[1 + i e^{-\operatorname{ArcCosh}[a x]}] - 96 a^2 \pi x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^2 \operatorname{Log}[1 + i e^{-\operatorname{ArcCosh}[a x]}] + \\
& 64 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^3 \operatorname{Log}[1 + i e^{-\operatorname{ArcCosh}[a x]}] + 48 i a^2 \pi^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x] \operatorname{Log}[1 - i e^{\operatorname{ArcCosh}[a x]}] + \\
& 96 a^2 \pi x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^2 \operatorname{Log}[1 - i e^{\operatorname{ArcCosh}[a x]}] - 8 a^2 \pi^3 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{Log}[1 + i e^{\operatorname{ArcCosh}[a x]}] - \\
& 64 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^3 \operatorname{Log}[1 + i e^{\operatorname{ArcCosh}[a x]}] + 8 a^2 \pi^3 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcCosh}[a x])\right]\right] + \\
& 48 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} (8 + \pi^2 - 4 i \pi \operatorname{ArcCosh}[a x] - 4 \operatorname{ArcCosh}[a x]^2) \operatorname{PolyLog}[2, -i e^{-\operatorname{ArcCosh}[a x]}] - \\
& 384 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{PolyLog}[2, i e^{-\operatorname{ArcCosh}[a x]}] - 192 i a^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x]^2 \operatorname{PolyLog}[2, -i e^{\operatorname{ArcCosh}[a x]}] + \\
& 48 i a^2 \pi^2 x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{PolyLog}[2, i e^{\operatorname{ArcCosh}[a x]}] + 192 a^2 \pi x^2 \sqrt{\frac{-1 + a x}{1 + a x}} \operatorname{ArcCosh}[a x] \operatorname{PolyLog}[2, i e^{\operatorname{ArcCosh}[a x]}] +
\end{aligned}$$

$$\begin{aligned}
& 192 a^2 \pi x^2 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[ax]}\right] - 384 i a^2 x^2 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[ax]}\right] + \\
& 384 i a^2 x^2 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[ax]}\right] - 192 a^2 \pi x^2 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[ax]}\right] - \\
& \left. 384 i a^2 x^2 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcCosh}[ax]}\right] - 384 i a^2 x^2 \sqrt{\frac{-1+ax}{1+ax}} \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[ax]}\right] \right)
\end{aligned}$$

Problem 326: Attempted integration timed out after 120 seconds.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^3 (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 9, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^3 (a+b \operatorname{ArcCosh}[cx])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 327: Attempted integration timed out after 120 seconds.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^4 (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 9, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^4 (a+b \operatorname{ArcCosh}[cx])^2}, x\right]$$

Result (type 1, 1 leaves):

???

### Problem 333: Attempted integration timed out after 120 seconds.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

### Problem 334: Attempted integration timed out after 120 seconds.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 106 leaves, 2 steps):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)^{3/2}}{b c x^4 (a + b \operatorname{ArcCosh}[c x])} - \frac{4 \sqrt{1 - c x} \operatorname{Unintegrable}\left[\frac{-1 + c^2 x^2}{x^5 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1 + c x}}$$

Result (type 1, 1 leaves):

???

### Problem 339: Attempted integration timed out after 120 seconds.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 160 leaves, 3 steps):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)^{5/2}}{b c x^2 (a + b \operatorname{ArcCosh}[c x])} + \frac{2 \sqrt{1 - c x} \operatorname{Unintegrable}\left[\frac{(-1 + c^2 x^2)^2}{x^3 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1 + c x}} + \frac{4 c \sqrt{1 - c x} \operatorname{Unintegrable}\left[\frac{(-1 + c^2 x^2)^2}{x (a + b \operatorname{ArcCosh}[c x])}, x\right]}{b \sqrt{-1 + c x}}$$

Result (type 1, 1 leaves):

???

**Problem 340: Attempted integration timed out after 120 seconds.**

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 341: Attempted integration timed out after 120 seconds.**

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(1 - c^2 x^2)^{5/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 489: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4 (a + b \operatorname{ArcCosh}[c x])}{d + e x^2} dx$$

Optimal (type 4, 627 leaves, 27 steps):

$$\begin{aligned}
& -\frac{a d x}{e^2} + \frac{b d \sqrt{-1+c x} \sqrt{1+c x}}{c e^2} - \frac{2 b \sqrt{-1+c x} \sqrt{1+c x}}{9 c^3 e} - \frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x}}{9 c e} \\
& \frac{b d x \operatorname{ArcCosh}[c x]}{e^2} + \frac{x^3 (a+b \operatorname{ArcCosh}[c x])}{3 e} + \frac{(-d)^{3/2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} \\
& \frac{(-d)^{3/2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} + \frac{(-d)^{3/2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} \\
& \frac{(-d)^{3/2} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} - \frac{b (-d)^{3/2} \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} + \\
& \frac{b (-d)^{3/2} \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} - \frac{b (-d)^{3/2} \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}} + \frac{b (-d)^{3/2} \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{5/2}}
\end{aligned}$$

Result (type 4, 956 leaves):

$$\begin{aligned}
& -\frac{a d x}{e^2} + \frac{a x^3}{3 e} + \frac{a d^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{e^{5/2}} + \\
& \frac{1}{4 e^{5/2}} b \left( \frac{4 d \sqrt{e} \left( \sqrt{\frac{-1+c x}{1+c x}} (1+c x) - c x \operatorname{ArcCosh}[c x] \right)}{c} - \frac{4 e^{3/2} \left( \sqrt{-1+c x} \sqrt{1+c x} (2+c^2 x^2) - 3 c^3 x^3 \operatorname{ArcCosh}[c x] \right)}{9 c^3} \right) + \\
& i d^{3/2} \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] -
\end{aligned}$$



$$\left. \begin{aligned}
& 2 \operatorname{PolyLog}\left[2, \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) - \\
& i d^{3/2} \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(c \sqrt{d} - i \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& 2 \operatorname{PolyLog}\left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) \right)
\end{aligned}$$

**Problem 490: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{d + e x^2} dx$$

Optimal (type 4, 521 leaves, 23 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{-1+c x} \sqrt{1+c x}}{4 c e} - \frac{b \operatorname{ArcCosh}[c x]}{4 c^2 e} + \frac{x^2 (a+b \operatorname{ArcCosh}[c x])}{2 e} + \frac{d (a+b \operatorname{ArcCosh}[c x])^2}{2 b e^2} - \frac{d (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} \\
& \frac{d (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} - \frac{d (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2} - \frac{d (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2} \\
& \frac{b d \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} - \frac{b d \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2} - \frac{b d \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2}
\end{aligned}$$

Result (type 4, 893 leaves):

$$\begin{aligned}
& \frac{1}{4 c^2 e^2} \left( 2 a c^2 e x^2 - 2 a c^2 d \operatorname{Log}[d + e x^2] + b \left( 2 c^2 e x^2 \operatorname{ArcCosh}[c x] - e \left( c x \sqrt{-1+c x} \sqrt{1+c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1+c x}}{\sqrt{2}}\right] \right) \right) - \right. \\
& c^2 d \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& c^2 d \left( \text{ArcCosh}[c x]^2 + 8 i \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \text{ArcCosh}[c x] \text{Log}\left[1 + \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \text{ArcCosh}[c x] \text{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& 2 \text{PolyLog}\left[2, -\frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \text{PolyLog}\left[2, \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] \Bigg)
\end{aligned}$$

**Problem 491: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 (a + b \text{ArcCosh}[c x])}{d + e x^2} dx$$

Optimal (type 4, 544 leaves, 23 steps):

$$\begin{aligned}
& \frac{a x}{e} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x}}{c e} + \frac{b x \text{ArcCosh}[c x]}{e} + \frac{\sqrt{-d} (a + b \text{ArcCosh}[c x]) \text{Log}\left[1 - \frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} - \\
& \frac{\sqrt{-d} (a + b \text{ArcCosh}[c x]) \text{Log}\left[1 + \frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} + \frac{\sqrt{-d} (a + b \text{ArcCosh}[c x]) \text{Log}\left[1 - \frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} - \\
& \frac{\sqrt{-d} (a + b \text{ArcCosh}[c x]) \text{Log}\left[1 + \frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} - \frac{b \sqrt{-d} \text{PolyLog}\left[2, -\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} + \\
& \frac{b \sqrt{-d} \text{PolyLog}\left[2, \frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} - \frac{b \sqrt{-d} \text{PolyLog}\left[2, -\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}} + \frac{b \sqrt{-d} \text{PolyLog}\left[2, \frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^{3/2}}
\end{aligned}$$

Result (type 4, 893 leaves):

$$\begin{aligned}
& \frac{a x}{e} - \frac{a \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{e^{3/2}} + b \left( \frac{-\sqrt{\frac{-1+cx}{1+cx}} (1+cx) + cx \operatorname{ArcCosh}[cx]}{ce} - \right. \\
& \frac{1}{4 e^{3/2}} i \sqrt{d} \left( \operatorname{ArcCosh}[cx]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d}+i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2 d+e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 - \frac{i(-c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(-c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] \right) + \\
& \frac{1}{4 e^{3/2}} i \sqrt{d} \left( \operatorname{ArcCosh}[cx]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d}-i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2 d+e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d}+\sqrt{c^2 d+e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] -
\end{aligned}$$

$$\left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i\left(-c\sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i\left(c\sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right] \right) \right)$$

**Problem 492: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x (a + b \operatorname{ArcCosh}[c x])}{d + e x^2} dx$$

Optimal (type 4, 449 leaves, 18 steps):

$$\begin{aligned} & -\frac{(a + b \operatorname{ArcCosh}[c x])^2}{2 b e} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e} + \\ & \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e} + \\ & \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e} \end{aligned}$$

Result (type 4, 808 leaves):

$$\begin{aligned}
& \frac{1}{2e} \left( b \operatorname{ArcCosh}[cx]^2 + 4i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} - i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d + e}}\right] + \right. \\
& 4i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} + i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d + e}}\right] + b \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 - \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& 2i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + b \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& 2i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + b \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + \\
& 2i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + b \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + \\
& 2i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + a \operatorname{Log}[d + ex^2] - \\
& b \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - b \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& \left. b \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - b \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] \right)
\end{aligned}$$

**Problem 493: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[cx]}{d + ex^2} dx$$

Optimal (type 4, 501 leaves, 18 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} + \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}}
\end{aligned}$$

Result (type 4, 821 leaves):

$$\begin{aligned}
& \frac{1}{2\sqrt{d}\sqrt{e}} \left( 2a \operatorname{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right] + 4b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d}-i\sqrt{e})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d+e}}\right] - \right. \\
& 4b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d}+i\sqrt{e})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d+e}}\right] + \\
& ib \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1-\frac{i(-c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 2b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i(-c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& ib \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1+\frac{i(-c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 2b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(-c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& ib \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1-\frac{i(c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 2b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i(c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + \\
& ib \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1+\frac{i(c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 2b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + \\
& ib \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - ib \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& \left. ib \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + ib \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d}+\sqrt{c^2d+e})e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] \right)
\end{aligned}$$

**Problem 494:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[cx]}{x(d + ex^2)} dx$$

Optimal (type 4, 489 leaves, 25 steps):



$$\begin{aligned}
& \frac{(a + b \operatorname{ArcCosh}[c x])^2}{b d} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d} \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d} \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d} - \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right]}{2 d} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d} \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d}
\end{aligned}$$

Result (type 4, 837 leaves):

$$\begin{aligned}
& -\frac{1}{2d} \left( 4i b \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{ArcTanh} \left[ \frac{(c\sqrt{d} - i\sqrt{e}) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[cx] \right]}{\sqrt{c^2d + e}} \right] + \right. \\
& 4i b \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{ArcTanh} \left[ \frac{(c\sqrt{d} + i\sqrt{e}) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[cx] \right]}{\sqrt{c^2d + e}} \right] - \\
& 2b \operatorname{ArcCosh}[cx] \operatorname{Log} \left[ 1 + e^{-2 \operatorname{ArcCosh}[cx]} \right] + b \operatorname{ArcCosh}[cx] \operatorname{Log} \left[ 1 - \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] - \\
& 2i b \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 - \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] + b \operatorname{ArcCosh}[cx] \operatorname{Log} \left[ 1 + \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] - \\
& 2i b \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] + b \operatorname{ArcCosh}[cx] \operatorname{Log} \left[ 1 - \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] + \\
& 2i b \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 - \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] + b \operatorname{ArcCosh}[cx] \operatorname{Log} \left[ 1 + \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] + \\
& 2i b \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}} \right] - 2a \operatorname{Log}[x] + a \operatorname{Log}[d + ex^2] + b \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcCosh}[cx]}] - \\
& b \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - b \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& \left. b \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - b \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] \right)
\end{aligned}$$

### Problem 495: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 (d + e x^2)} dx$$

Optimal (type 4, 543 leaves, 23 steps):

$$\begin{aligned} & -\frac{a + b \operatorname{ArcCosh}[c x]}{d x} + \frac{b c \operatorname{ArcTan}\left[\frac{\sqrt{-1 + c x} \sqrt{1 + c x}}{d}\right]}{d} + \frac{\sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} - \\ & \frac{\sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} + \frac{\sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} - \\ & \frac{\sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} - \frac{b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} + \\ & \frac{b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} - \frac{b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} + \frac{b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 (-d)^{3/2}} \end{aligned}$$

Result (type 4, 887 leaves):

$$\begin{aligned} & \frac{1}{4 d^{3/2} x} \left( -4 a \sqrt{d} - 4 a \sqrt{e} x \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - 4 b \sqrt{d} \left( \operatorname{ArcCosh}[c x] + c x \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1 + c x} \sqrt{1 + c x}}\right] \right) \right) - \\ & i b \sqrt{e} x \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\ & 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) + \\
& i b \sqrt{e} x \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} - i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \right. \\
& \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \right. \\
& \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) \right)
\end{aligned}$$

**Problem 496:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^3 (d + e x^2)} dx$$

Optimal (type 4, 550 leaves, 27 steps):

$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{2 d x} - \frac{a+b \operatorname{ArcCosh}[c x]}{2 d x^2} - \frac{e(a+b \operatorname{ArcCosh}[c x])^2}{b d^2} - \frac{e(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d^2} + \\
& \frac{e(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2} + \frac{e(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2} + \\
& \frac{e(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2} + \frac{e(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2} + \frac{b e \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcCosh}[c x]}\right]}{2 d^2} + \\
& \frac{b e \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2} + \frac{b e \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2} + \frac{b e \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2} + \frac{b e \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2}
\end{aligned}$$

Result (type 4, 913 leaves):

$$\begin{aligned}
& \frac{1}{4 d^2 x^2} \left( -2 a d - 4 a e x^2 \operatorname{Log}[x] + 2 a e x^2 \operatorname{Log}[d+e x^2] + \right. \\
& \left. b \left( 2 d \left( c x \sqrt{-1+c x} \sqrt{1+c x} - \operatorname{ArcCosh}[c x] \right) - 2 e x^2 \left( \operatorname{ArcCosh}[c x] \left( \operatorname{ArcCosh}[c x] + 2 \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c x]}\right] \right) - \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcCosh}[c x]}\right] \right) + \right. \\
& \left. e x^2 \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(c \sqrt{d}+i \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right] + \right. \\
& \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1-\frac{i\left(-c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(-c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \right. \\
& \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1+\frac{i\left(c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \right.
\end{aligned}$$

$$\left. \begin{aligned}
& 2 \operatorname{PolyLog}\left[2, \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right\} + \\
& e x^2 \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(c \sqrt{d} - i \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) \right) \right)
\end{aligned}
\right)$$

**Problem 497: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d + e x^2)} dx$$

Optimal (type 4, 624 leaves, 28 steps):

$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{6 d x^2} - \frac{a+b \operatorname{ArcCosh}[c x]}{3 d x^3} + \frac{e(a+b \operatorname{ArcCosh}[c x])}{d^2 x} + \frac{b c^3 \operatorname{ArcTan}\left[\frac{\sqrt{-1+c x} \sqrt{1+c x}}{6 d}\right]}{6 d} - \\
& \frac{b c e \operatorname{ArcTan}\left[\frac{\sqrt{-1+c x} \sqrt{1+c x}}{d^2}\right]}{d^2} + \frac{e^{3/2}(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} - \frac{e^{3/2}(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} + \\
& \frac{e^{3/2}(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} - \frac{e^{3/2}(a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} - \frac{b e^{3/2} \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} + \\
& \frac{b e^{3/2} \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} - \frac{b e^{3/2} \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}} + \frac{b e^{3/2} \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2(-d)^{5/2}}
\end{aligned}$$

Result (type 4, 972 leaves):

$$\begin{aligned}
& \frac{1}{12 d^{5/2} x^3} \left( -4 a d^{3/2} + 12 a \sqrt{d} e x^2 + 12 a e^{3/2} x^3 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + b \left( 12 \sqrt{d} e x^2 \left( \operatorname{ArcCosh}[c x] + c x \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1+c x} \sqrt{1+c x}}\right] \right) + \right. \right. \\
& \left. \left. 2 d^{3/2} \left( c x \sqrt{-1+c x} \sqrt{1+c x} - 2 \operatorname{ArcCosh}[c x] - c^3 x^3 \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1+c x} \sqrt{1+c x}}\right] \right) \right) + \right. \\
& \left. 3 i e^{3/2} x^3 \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \right. \\
& \left. \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) + \right. \\
& \left. \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) - \right. \\
& \left. \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) \right)
\end{aligned}$$

$$\left. \begin{aligned}
& 2 \operatorname{PolyLog}\left[2, \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right\} - \\
& 3 i e^{3/2} x^3 \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(c \sqrt{d} - i \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) \right) \right)
\end{aligned} \right)$$

**Problem 498:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 562 leaves, 24 steps):



$$\begin{aligned}
& \frac{d (a + b \operatorname{ArcCosh}[c x])}{2 e^2 (d + e x^2)} - \frac{(a + b \operatorname{ArcCosh}[c x])^2}{2 b e^2} - \frac{b c \sqrt{d} \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{2 e^2 \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2} + \\
& \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^2}
\end{aligned}$$

Result (type 4, 1108 leaves):

$$\begin{aligned}
& \frac{1}{4 e^2} \left( \frac{2 a d}{d + e x^2} + 2 a \operatorname{Log}[d + e x^2] + \right. \\
& b \left( \frac{\sqrt{d} \operatorname{ArcCosh}[c x]}{\sqrt{d} - i \sqrt{e} x} + \frac{\sqrt{d} \operatorname{ArcCosh}[c x]}{\sqrt{d} + i \sqrt{e} x} + 2 \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} - i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \frac{i c \sqrt{d} \operatorname{Log}\left[\frac{2 e \left(i \sqrt{e} + c^2 \sqrt{d} x - i \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x}\right)}{c \sqrt{-c^2 d - e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} + \frac{i c \sqrt{d} \operatorname{Log}\left[\frac{2 e \left(-\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x}\right)}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} - \\
& 2 \operatorname{PolyLog}\left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) \right) \right)
\end{aligned}$$

Problem 500: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x (d + e x^2)^2} dx$$

Optimal (type 4, 598 leaves, 29 steps):

$$\begin{aligned}
& \frac{a + b \operatorname{ArcCosh}[c x]}{2 d (d + e x^2)} + \frac{(a + b \operatorname{ArcCosh}[c x])^2}{b d^2} - \frac{b c \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{2 d^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d^2} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^2} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^2} - \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^2} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^2} - \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right]}{2 d^2} - \\
& \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^2} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^2} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^2} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^2}
\end{aligned}$$

Result (type 4, 1146 leaves):

$$\begin{aligned}
& \frac{a}{2d^2 + 2de x^2} + \frac{a \operatorname{Log}[x]}{d^2} - \frac{a \operatorname{Log}[d + ex^2]}{2d^2} + \\
& \frac{1}{4d^2} b \left( \frac{\sqrt{d} \operatorname{ArcCosh}[cx]}{\sqrt{d} - i\sqrt{e}x} + \frac{\sqrt{d} \operatorname{ArcCosh}[cx]}{\sqrt{d} + i\sqrt{e}x} - 8i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} - i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d + e}}\right] - \right. \\
& 8i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} + i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d + e}}\right] + 4 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[cx]}\right] - \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 - \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& 2 \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ic\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] - \\
& \frac{i c \sqrt{d} \operatorname{Log}\left[\frac{2e\left(i\sqrt{e} + c^2\sqrt{d}x - i\sqrt{-c^2d - e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d - e}(\sqrt{d} + i\sqrt{e}x)}\right]}{\sqrt{-c^2d - e}} + \frac{i c \sqrt{d} \operatorname{Log}\left[\frac{2e\left(-\sqrt{e} - ic^2\sqrt{d}x + \sqrt{-c^2d - e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d - e}(i\sqrt{d} + \sqrt{e}x)}\right]}{\sqrt{-c^2d - e}} - \\
& 2 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[cx]}\right] + 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] + 2 \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2d + e}) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{e}}\right] +
\end{aligned}$$

$$\left. 2 \operatorname{PolyLog}\left[2, -\frac{i\left(c\sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 2 \operatorname{PolyLog}\left[2, \frac{i\left(c\sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)$$

**Problem 501: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^3 (d + e x^2)^2} dx$$

Optimal (type 4, 634 leaves, 31 steps):

$$\begin{aligned} & \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x}}{2 d^2 x} - \frac{a + b \operatorname{ArcCosh}[c x]}{2 d^2 x^2} - \frac{e (a + b \operatorname{ArcCosh}[c x])}{2 d^2 (d + e x^2)} - \frac{2 e (a + b \operatorname{ArcCosh}[c x])^2}{b d^3} + \frac{b c e \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{2 d^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x}} \\ & \frac{2 e (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d^3} + \frac{e (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{d^3} + \frac{e (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{d^3} \\ & \frac{e (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{d^3} + \frac{e (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{d^3} + \frac{b e \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d^3} + \\ & \frac{b e \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{d^3} + \frac{b e \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{d^3} + \frac{b e \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{d^3} + \frac{b e \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{d^3} \end{aligned}$$

Result (type 4, 1237 leaves):

$$-\frac{a}{2 d^2 x^2} - \frac{a e}{2 d^2 (d + e x^2)} - \frac{2 a e \operatorname{Log}[x]}{d^3} + \frac{a e \operatorname{Log}[d + e x^2]}{d^3} +$$

$$\begin{aligned}
& b \left( \frac{c x \sqrt{-1+c x} \sqrt{1+c x} - \operatorname{ArcCosh}[c x]}{2 d^2 x^2} + \frac{i e \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e^{i \sqrt{e} - c^2 \sqrt{d} x - i \sqrt{-c^2 d - e} \sqrt{-1+c x} \sqrt{1+c x}}}{c \sqrt{-c^2 d - e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right)}{4 d^{5/2}} \right) + \\
& i e \left( \frac{-\frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} - \frac{c \operatorname{Log}\left[\frac{2 e^{-\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1+c x} \sqrt{1+c x}}}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}}}{4 d^{5/2}} \right) - \\
& \frac{e \left( \operatorname{ArcCosh}[c x] \left( \operatorname{ArcCosh}[c x] + 2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right] \right) - \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right] \right)}{d^3} + \\
& \frac{1}{2 d^3} e \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2 d^3} e \left( \text{ArcCosh}[c x]^2 + 8 i \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \text{ArcCosh}[c x] \text{Log}\left[1 + \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \text{ArcCosh}[c x] \text{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& 2 \text{PolyLog}\left[2, -\frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \text{PolyLog}\left[2, \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] \Bigg)
\end{aligned}$$

**Problem 502: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4 (a + b \text{ArcCosh}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 839 leaves, 49 steps):

$$\begin{aligned}
& \frac{a x}{e^2} - \frac{b \sqrt{-1+c x} \sqrt{1+c x}}{c e^2} + \frac{b x \operatorname{ArcCosh}[c x]}{e^2} - \frac{d (a+b \operatorname{ArcCosh}[c x])}{4 e^{5/2} (\sqrt{-d}-\sqrt{e} x)} + \frac{d (a+b \operatorname{ArcCosh}[c x])}{4 e^{5/2} (\sqrt{-d}+\sqrt{e} x)} + \frac{b c d \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}\right]}{2 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}} e^{5/2}} - \\
& \frac{b c d \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}\right]}{2 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}} e^{5/2}} + \frac{3 \sqrt{-d} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} - \frac{3 \sqrt{-d} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} + \\
& \frac{3 \sqrt{-d} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} - \frac{3 \sqrt{-d} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} - \frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} + \\
& \frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} - \frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}} + \frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}
\end{aligned}$$

Result (type 4, 1185 leaves):

$$\begin{aligned}
& \frac{1}{8 e^{5/2}} \left( 8 a \sqrt{e} x + \frac{4 a d \sqrt{e} x}{d+e x^2} - 12 a \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + b \left( \frac{8 \sqrt{e} \left( -\sqrt{\frac{-1+c x}{1+c x}} (1+c x) + c x \operatorname{ArcCosh}[c x] \right)}{c} \right) + \right. \\
& \left. 2 d \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e \left( i \sqrt{e} + c^2 \sqrt{d} x - i \sqrt{-c^2 d-e} \sqrt{-1+c x} \sqrt{1+c x} \right)}{c \sqrt{-c^2 d-e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d-e}} \right) + 2 d \left( \frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e \left( -\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d-e} \sqrt{-1+c x} \sqrt{1+c x} \right)}{c \sqrt{-c^2 d-e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d-e}} \right) - \right. \\
& \left. 3 i \sqrt{d} \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right] \right) + \right. \\
& \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1-\frac{i \left( -c \sqrt{d} + \sqrt{c^2 d+e} \right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i \left( -c \sqrt{d} + \sqrt{c^2 d+e} \right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) + \\
& 3 i \sqrt{d} \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} - i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) \right)
\end{aligned}$$

**Problem 503: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 792 leaves, 46 steps):



$$\begin{aligned}
& \frac{a + b \operatorname{ArcCosh}[c x]}{4 e^{3/2} (\sqrt{-d} - \sqrt{e} x)} - \frac{a + b \operatorname{ArcCosh}[c x]}{4 e^{3/2} (\sqrt{-d} + \sqrt{e} x)} - \frac{b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d} + \sqrt{e}} \sqrt{-1+c x}}\right]}{2 \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{c \sqrt{-d} + \sqrt{e}} e^{3/2}} + \\
& \frac{b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d} + \sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{-1+c x}}\right]}{2 \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{c \sqrt{-d} + \sqrt{e}} e^{3/2}} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} + \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 \sqrt{-d} e^{3/2}}
\end{aligned}$$

Result (type 4, 1130 leaves):

$$\begin{aligned}
& \frac{1}{8 e^{3/2}} \left( -\frac{4 a \sqrt{e} x}{d + e x^2} + \frac{4 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} + \right. \\
& \left. b \left( -\frac{2 \operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} - 2 \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e \left( i \sqrt{e} + c^2 \sqrt{d} x - i \sqrt{-c^2 d - e} \sqrt{-1+c x} \sqrt{1+c x} \right)}{c \sqrt{-c^2 d - e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right) - \frac{2 c \operatorname{Log}\left[\frac{2 e \left( -\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1+c x} \sqrt{1+c x} \right)}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} + \right. \\
& \left. \frac{1}{\sqrt{d}} i \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& \left. \left. 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) - \\
& \frac{1}{\sqrt{d}} i \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c\sqrt{d} - i\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) \right)
\end{aligned}$$

**Problem 504: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d + e x^2)^2} dx$$

Optimal (type 4, 804 leaves, 26 steps):

$$\begin{aligned}
& - \frac{a + b \operatorname{ArcCosh}[c x]}{4 d \sqrt{e} (\sqrt{-d} - \sqrt{e} x)} + \frac{a + b \operatorname{ArcCosh}[c x]}{4 d \sqrt{e} (\sqrt{-d} + \sqrt{e} x)} + \frac{b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{1 + c x}}{\sqrt{c \sqrt{-d} + \sqrt{e}} \sqrt{-1 + c x}}\right]}{2 d \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{c \sqrt{-d} + \sqrt{e}} \sqrt{e}} - \\
& \frac{b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d} + \sqrt{e}} \sqrt{1 + c x}}{\sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{-1 + c x}}\right]}{2 d \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{c \sqrt{-d} + \sqrt{e}} \sqrt{e}} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{3/2} \sqrt{e}}
\end{aligned}$$

Result (type 4, 1126 leaves):

$$\begin{aligned}
& \frac{1}{2} \left( \frac{a x}{d^2 + d e x^2} + \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{d^{3/2} \sqrt{e}} + \right. \\
& \frac{1}{2 d^{3/2} \sqrt{e}} b \left( \frac{\sqrt{d} \operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{\sqrt{d} \operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} - i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] - \right. \\
& \left. 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& \left. i \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& i \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& i \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& i \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& \frac{c\sqrt{d} \operatorname{Log}\left[\frac{2e(i\sqrt{e} + c^2\sqrt{d}x - i\sqrt{-c^2 d - e}\sqrt{-1 + cx}\sqrt{1 + cx})}{c\sqrt{-c^2 d - e}(\sqrt{d} + i\sqrt{e}x)}\right]}{\sqrt{-c^2 d - e}} + \frac{c\sqrt{d} \operatorname{Log}\left[\frac{2e(-\sqrt{e} - i c^2\sqrt{d}x + \sqrt{-c^2 d - e}\sqrt{-1 + cx}\sqrt{1 + cx})}{c\sqrt{-c^2 d - e}(i\sqrt{d} + \sqrt{e}x)}\right]}{\sqrt{-c^2 d - e}} + \\
& i \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - i \operatorname{PolyLog}\left[2, \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& i \operatorname{PolyLog}\left[2, -\frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + i \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \Bigg)
\end{aligned}$$

Problem 505: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 (d + e x^2)^2} dx$$

Optimal (type 4, 846 leaves, 49 steps):

$$\begin{aligned}
& -\frac{a + b \operatorname{ArcCosh}[c x]}{d^2 x} + \frac{\sqrt{e} (a + b \operatorname{ArcCosh}[c x])}{4 d^2 (\sqrt{-d} - \sqrt{e} x)} - \frac{\sqrt{e} (a + b \operatorname{ArcCosh}[c x])}{4 d^2 (\sqrt{-d} + \sqrt{e} x)} + \frac{b c \operatorname{ArcTan}\left[\frac{\sqrt{-1+c x} \sqrt{1+c x}}{d}\right]}{d^2} - \\
& \frac{b c \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}\right]}{2 d^2 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}}} + \frac{b c \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}\right]}{2 d^2 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}}} - \frac{3 \sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} + \\
& \frac{3 \sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} - \frac{3 \sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} + \\
& \frac{3 \sqrt{e} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} + \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} - \\
& \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} + \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}} - \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{4 (-d)^{5/2}}
\end{aligned}$$

Result (type 4, 1203 leaves):

$$\begin{aligned}
& \frac{1}{8 d^{5/2}} \left( -\frac{8 a \sqrt{d}}{x} - \frac{4 a \sqrt{d} e x}{d + e x^2} - 12 a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + \right. \\
& b \left( -\frac{8 \sqrt{d} (\operatorname{ArcCosh}[c x] + c x \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1+c x} \sqrt{1+c x}}\right])}{x} - 2 \sqrt{d} \sqrt{e} \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e (i \sqrt{e} + c^2 \sqrt{d} x - i \sqrt{-c^2 d - e} \sqrt{-1+c x} \sqrt{1+c x})}{c \sqrt{-c^2 d - e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right) + \right. \\
& \left. \left. 2 \sqrt{d} \sqrt{e} \left( -\frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} - \frac{c \operatorname{Log}\left[\frac{2 e (-\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1+c x} \sqrt{1+c x})}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right) \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 i \sqrt{e} \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) + \\
& 3 i \sqrt{e} \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c \sqrt{d} - i \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \right. \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

Problem 506: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCosh}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 737 leaves, 29 steps):

$$\begin{aligned} & \frac{b c d x (1 - c^2 x^2)}{8 e^2 (c^2 d + e) \sqrt{-1 + c x} \sqrt{1 + c x} (d + e x^2)} - \frac{d^2 (a + b \operatorname{ArcCosh}[c x])}{4 e^3 (d + e x^2)^2} + \frac{d (a + b \operatorname{ArcCosh}[c x])}{e^3 (d + e x^2)} - \frac{(a + b \operatorname{ArcCosh}[c x])^2}{2 b e^3} - \\ & \frac{b c \sqrt{d} \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{e^3 \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c \sqrt{d} (2 c^2 d + e) \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{8 e^3 (c^2 d + e)^{3/2} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^3} + \\ & \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^3} + \\ & \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 e^3} \end{aligned}$$

Result (type 4, 1564 leaves):

$$\begin{aligned} & -\frac{a d^2}{4 e^3 (d + e x^2)^2} + \frac{a d}{e^3 (d + e x^2)} + \frac{a \operatorname{Log}[d + e x^2]}{2 e^3} + \\ & b \left( \frac{7 i \sqrt{d} \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e \left( i \sqrt{e} + c^2 \sqrt{d} x - i \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x} \right)}{c \sqrt{-c^2 d - e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right)}{16 e^3} - \frac{7 i \sqrt{d} \left( -\frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} - \frac{c \operatorname{Log}\left[\frac{2 e \left( -\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x} \right)}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right)}{16 e^3} - \frac{1}{16 e^{5/2}} \right) \end{aligned}$$

$$d \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d+i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right) -$$

$$d \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} - \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} + c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d-i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right) +$$

16 e<sup>5/2</sup>

$$\frac{1}{4 e^3} \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} + i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] \right) +$$

$$2 \text{ArcCosh}[cx] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] +$$

$$2 \text{ArcCosh}[cx] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] -$$

$$2 \text{PolyLog} \left[ 2, \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 2 \text{PolyLog} \left[ 2, -\frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] \right) +$$

$$\frac{1}{4 e^3} \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] \right) +$$



$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)\right)
\end{aligned}$$

**Problem 509:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x (d + e x^2)^3} dx$$

Optimal (type 4, 772 leaves, 34 steps):

$$\begin{aligned}
 & - \frac{b c e x (1 - c^2 x^2)}{8 d^2 (c^2 d + e) \sqrt{-1 + c x} \sqrt{1 + c x} (d + e x^2)} + \frac{a + b \operatorname{ArcCosh}[c x]}{4 d (d + e x^2)^2} + \frac{a + b \operatorname{ArcCosh}[c x]}{2 d^2 (d + e x^2)} + \\
 & \frac{(a + b \operatorname{ArcCosh}[c x])^2}{b d^3} - \frac{b c \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{2 d^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c (2 c^2 d + e) \sqrt{-1 + c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e} x}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{8 d^{5/2} (c^2 d + e)^{3/2} \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
 & \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d^3} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^3} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^3} \\
 & \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^3} - \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^3} - \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right]}{2 d^3} \\
 & \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^3} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 d^3} - \frac{b \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^3} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 d^3}
 \end{aligned}$$

Result (type 4, 1613 leaves):

$$\begin{aligned}
 & \frac{a}{4 d (d + e x^2)^2} + \frac{a}{2 d^2 (d + e x^2)} + \frac{a \operatorname{Log}[x]}{d^3} - \frac{a \operatorname{Log}[d + e x^2]}{2 d^3} + \\
 & b \left( \frac{5 i \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e \left( i \sqrt{e} + c^2 \sqrt{d} x - i \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x} \right)}{c \sqrt{-c^2 d - e} (\sqrt{d} + i \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right)}{16 d^{5/2}} - \frac{5 i \left( -\frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} - \frac{c \operatorname{Log}\left[\frac{2 e \left( -i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x} \right)}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right)}{16 d^{5/2}} + \frac{1}{16 d^2} \right) \\
 & \sqrt{e} \left( \frac{c \sqrt{-1 + c x} \sqrt{1 + c x}}{(c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\operatorname{ArcCosh}[c x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{c^3 \sqrt{d} \left( \operatorname{Log}[4] + \operatorname{Log}\left[\frac{e \sqrt{c^2 d + e} (-i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x})}{c^3 (d + i \sqrt{d} \sqrt{e} x)}\right] \right)}{\sqrt{e} (c^2 d + e)^{3/2}} \right) +
 \end{aligned}$$

$$\frac{1}{16 d^2} \sqrt{e} \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} - \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} + c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d-i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right) +$$

$$\frac{\text{ArcCosh}[cx] (\text{ArcCosh}[cx] + 2 \text{Log}[1 + e^{-2 \text{ArcCosh}[cx]}]) - \text{PolyLog}[2, -e^{-2 \text{ArcCosh}[cx]}]}{2 d^3} -$$

$$\frac{1}{4 d^3} \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} + i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] + \right.$$

$$2 \text{ArcCosh}[cx] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] +$$

$$2 \text{ArcCosh}[cx] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] -$$

$$2 \text{PolyLog}[2, \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}}] - 2 \text{PolyLog}[2, -\frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}}] \left. \right) -$$

$$\frac{1}{4 d^3} \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] + \right.$$

$$2 \text{ArcCosh}[cx] \text{Log} \left[ 1 + \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 4 i \text{ArcSin} \left[ \frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 + \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] +$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right] \right) \right)
\end{aligned}$$

Problem 510: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^3 (d + e x^2)^3} dx$$

Optimal (type 4, 834 leaves, 36 steps):

$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{2 d^3 x} + \frac{b c e^2 x (1-c^2 x^2)}{8 d^3 (c^2 d+e) \sqrt{-1+c x} \sqrt{1+c x} (d+e x^2)} - \frac{a+b \operatorname{ArcCosh}[c x]}{2 d^3 x^2} - \frac{e (a+b \operatorname{ArcCosh}[c x])}{4 d^2 (d+e x^2)^2} - \frac{e (a+b \operatorname{ArcCosh}[c x])}{d^3 (d+e x^2)} - \\
& \frac{3 e (a+b \operatorname{ArcCosh}[c x])^2}{b d^4} + \frac{b c e \sqrt{-1+c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d+e} x}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{d^{7/2} \sqrt{c^2 d+e} \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c e (2 c^2 d+e) \sqrt{-1+c^2 x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d+e} x}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{8 d^{7/2} (c^2 d+e)^{3/2} \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{3 e (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c x]}\right]}{d^4} + \frac{3 e (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^4} + \\
& \frac{3 e (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^4} + \frac{3 e (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^4} + \\
& \frac{3 e (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^4} + \frac{3 b e \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcCosh}[c x]}\right]}{2 d^4} + \frac{3 b e \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^4} + \\
& \frac{3 b e \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^4} + \frac{3 b e \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^4} + \frac{3 b e \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^4}
\end{aligned}$$

Result (type 4, 1670 leaves):

$$\begin{aligned}
& -\frac{a}{2 d^3 x^2} - \frac{a e}{4 d^2 (d+e x^2)^2} - \frac{a e}{d^3 (d+e x^2)} - \frac{3 a e \operatorname{Log}[x]}{d^4} + \frac{3 a e \operatorname{Log}[d+e x^2]}{2 d^4} + \\
& b \left( \frac{c x \sqrt{-1+c x} \sqrt{1+c x} - \operatorname{ArcCosh}[c x]}{2 d^3 x^2} + \frac{9 i e \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d}+\sqrt{e} x} + \frac{c \operatorname{Log}\left[ \frac{2 e \left( i \sqrt{e}+c^2 \sqrt{d} x-i \sqrt{-c^2 d-e} \sqrt{-1+c x} \sqrt{1+c x} \right)}{c \sqrt{-c^2 d-e} (\sqrt{d+i} \sqrt{e} x)} \right]}{\sqrt{-c^2 d-e}} \right)}{16 d^{7/2}} \right) +
\end{aligned}$$

$$\frac{9 i e \left( -\frac{\text{ArcCosh}[c x]}{i \sqrt{d} + \sqrt{e} x} - \frac{c \text{Log}\left[\frac{2e^{-\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{-c^2 d - e} \sqrt{-1 + c x} \sqrt{1 + c x}}}{c \sqrt{-c^2 d - e} (i \sqrt{d} + \sqrt{e} x)}\right]}{\sqrt{-c^2 d - e}} \right)}{16 d^{7/2}} - \frac{1}{16 d^3}$$

$$e^{3/2} \left( \frac{c \sqrt{-1 + c x} \sqrt{1 + c x}}{(c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[c x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[\frac{e \sqrt{c^2 d + e} (-i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x})}{c^3 (d + i \sqrt{d} \sqrt{e} x)}\right] \right)}{\sqrt{e} (c^2 d + e)^{3/2}} \right) -$$

$$\frac{1}{16 d^3} e^{3/2} \left( \frac{c \sqrt{-1 + c x} \sqrt{1 + c x}}{(c^2 d + e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[c x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} - \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[\frac{e \sqrt{c^2 d + e} (-i \sqrt{e} + c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{-1 + c x} \sqrt{1 + c x})}{c^3 (d - i \sqrt{d} \sqrt{e} x)}\right] \right)}{\sqrt{e} (c^2 d + e)^{3/2}} \right) -$$

$$\frac{3 e (\text{ArcCosh}[c x] (\text{ArcCosh}[c x] + 2 \text{Log}[1 + e^{-2 \text{ArcCosh}[c x]}]) - \text{PolyLog}[2, -e^{-2 \text{ArcCosh}[c x]}])}{2 d^4} +$$

$$\frac{1}{4 d^4} 3 e \left( \text{ArcCosh}[c x]^2 + 8 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(c \sqrt{d} + i \sqrt{e}) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) +$$

$$2 \text{ArcCosh}[c x] \text{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] +$$

$$2 \text{ArcCosh}[c x] \text{Log}\left[1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d + e}) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] -$$

$$\left. \begin{aligned}
& 2 \operatorname{PolyLog}\left[2, \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right\} + \\
& \frac{1}{4 d^4} 3 e \left( \operatorname{ArcCosh}[c x]^2 + 8 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(c \sqrt{d} - i \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& 2 \operatorname{PolyLog}\left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] \right\}
\end{aligned}$$

**Problem 511:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 (a + b \operatorname{ArcCosh}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 1224 leaves, 80 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{-d} \sqrt{-1+c x} \sqrt{1+c x}}{16 e^2 (c^2 d+e) (\sqrt{-d}-\sqrt{e} x)} - \frac{b c \sqrt{-d} \sqrt{-1+c x} \sqrt{1+c x}}{16 e^2 (c^2 d+e) (\sqrt{-d}+\sqrt{e} x)} - \frac{\sqrt{-d} (a+b \operatorname{ArcCosh}[c x])}{16 e^{5/2} (\sqrt{-d}-\sqrt{e} x)^2} + \frac{5 (a+b \operatorname{ArcCosh}[c x])}{16 e^{5/2} (\sqrt{-d}-\sqrt{e} x)} + \\
& \frac{\sqrt{-d} (a+b \operatorname{ArcCosh}[c x])}{16 e^{5/2} (\sqrt{-d}+\sqrt{e} x)^2} - \frac{5 (a+b \operatorname{ArcCosh}[c x])}{16 e^{5/2} (\sqrt{-d}+\sqrt{e} x)} - \frac{b c^3 d \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{-1+c x}}\right]}{8 (c \sqrt{-d}-\sqrt{e})^{3/2} (c \sqrt{-d}+\sqrt{e})^{3/2} e^{5/2}} - \frac{5 b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{-1+c x}}\right]}{8 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}} e^{5/2}} + \\
& \frac{b c^3 d \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{-1+c x}}\right]}{8 (c \sqrt{-d}-\sqrt{e})^{3/2} (c \sqrt{-d}+\sqrt{e})^{3/2} e^{5/2}} + \frac{5 b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{-1+c x}}\right]}{8 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}} e^{5/2}} + \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} - \\
& \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} + \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} - \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} - \\
& \frac{3 b \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} + \frac{3 b \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} - \frac{3 b \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}} + \frac{3 b \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 \sqrt{-d} e^{5/2}}
\end{aligned}$$

Result (type 4, 1594 leaves):

$$\begin{aligned}
& \frac{a d x}{4 e^2 (d+e x^2)^2} - \frac{5 a x}{8 e^2 (d+e x^2)} + \frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 \sqrt{d} e^{5/2}} + \\
& b \left( \frac{5 \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d}+\sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e^{i \sqrt{e}+c^2 \sqrt{d} x-i \sqrt{-c^2 d-e}} \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{-c^2 d-e} (\sqrt{d}+i \sqrt{e} x)}\right]}{\sqrt{-c^2 d-e}} \right)}{16 e^{5/2}} + \frac{5 \left( \frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d}+\sqrt{e} x} - \frac{c \operatorname{Log}\left[\frac{2 e^{-i \sqrt{e}-i c^2 \sqrt{d} x+i \sqrt{-c^2 d-e}} \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{-c^2 d-e} (i \sqrt{d}+\sqrt{e} x)}\right]}{\sqrt{-c^2 d-e}} \right)}{16 e^{5/2}} \right) + \frac{1}{16 e^2}
\end{aligned}$$



$$\begin{aligned}
& i \sqrt{d} \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d+i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right) - \\
& \frac{1}{16 e^2} i \sqrt{d} \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} - \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} + c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d-i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right) + \\
& \frac{1}{32 \sqrt{d} e^{5/2}} 3 i \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin}\left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh}\left[ \frac{(c \sqrt{d} + i \sqrt{e}) \text{Tanh}\left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] + \right. \\
& 2 \text{ArcCosh}[cx] \text{Log}\left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 4 i \text{ArcSin}\left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log}\left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + \\
& 2 \text{ArcCosh}[cx] \text{Log}\left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + 4 i \text{ArcSin}\left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log}\left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - \\
& \left. 2 \text{PolyLog}\left[ 2, \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 2 \text{PolyLog}\left[ 2, -\frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] \right) - \\
& \frac{1}{32 \sqrt{d} e^{5/2}} 3 i \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin}\left[ \frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh}\left[ \frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh}\left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) \right) \right)
\end{aligned}$$

**Problem 512:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 1234 leaves, 62 steps):

$$\begin{aligned}
& - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e(c^2d+e)(\sqrt{-d}-\sqrt{e}x)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e(c^2d+e)(\sqrt{-d}+\sqrt{e}x)} - \frac{a+b\text{ArcCosh}[cx]}{16\sqrt{-d}e^{3/2}(\sqrt{-d}-\sqrt{e}x)^2} - \frac{a+b\text{ArcCosh}[cx]}{16de^{3/2}(\sqrt{-d}-\sqrt{e}x)} + \\
& \frac{a+b\text{ArcCosh}[cx]}{16\sqrt{-d}e^{3/2}(\sqrt{-d}+\sqrt{e}x)^2} + \frac{a+b\text{ArcCosh}[cx]}{16de^{3/2}(\sqrt{-d}+\sqrt{e}x)} + \frac{bc^3\text{ArcTanh}\left[\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right]}{8(c\sqrt{-d}-\sqrt{e})^{3/2}(c\sqrt{-d}+\sqrt{e})^{3/2}e^{3/2}} + \frac{bc\text{ArcTanh}\left[\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right]}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} - \\
& \frac{bc^3\text{ArcTanh}\left[\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{-1+cx}}\right]}{8(c\sqrt{-d}-\sqrt{e})^{3/2}(c\sqrt{-d}+\sqrt{e})^{3/2}e^{3/2}} - \frac{bc\text{ArcTanh}\left[\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{-1+cx}}\right]}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} - \frac{(a+b\text{ArcCosh}[cx])\text{Log}\left[1-\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} + \\
& \frac{(a+b\text{ArcCosh}[cx])\text{Log}\left[1+\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\text{ArcCosh}[cx])\text{Log}\left[1-\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\text{ArcCosh}[cx])\text{Log}\left[1+\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} + \\
& \frac{b\text{PolyLog}\left[2,-\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} - \frac{b\text{PolyLog}\left[2,\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} + \frac{b\text{PolyLog}\left[2,-\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}} - \frac{b\text{PolyLog}\left[2,\frac{\sqrt{e}e^{\text{ArcCosh}[cx]}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{16(-d)^{3/2}e^{3/2}}
\end{aligned}$$

Result (type 4, 1602 leaves):

$$\begin{aligned}
& - \frac{ax}{4e(d+ex^2)^2} + \frac{ax}{8de(d+ex^2)} + \frac{a\text{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}{8d^{3/2}e^{3/2}} + \\
& b \left( \frac{\frac{\text{ArcCosh}[cx]}{-i\sqrt{d}+\sqrt{e}x} + \frac{c\text{Log}\left[\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}x-i\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right]}{\sqrt{-c^2d-e}}}{16de^{3/2}} - \frac{\frac{\text{ArcCosh}[cx]}{i\sqrt{d}+\sqrt{e}x} - \frac{c\text{Log}\left[\frac{2e\left(-\sqrt{e}-i\sqrt{e}c^2\sqrt{d}x+\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(i\sqrt{d}+\sqrt{e}x\right)}\right]}{\sqrt{-c^2d-e}}}{16de^{3/2}} - \frac{1}{16\sqrt{d}e} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{i \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d+i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right)}{16 \sqrt{d} e} + \right. \\
& \left. \frac{i \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} - \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} + c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx})}{c^3 (d-i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right)}{16 \sqrt{d} e} + \right. \\
& \frac{1}{32 d^{3/2} e^{3/2}} i \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} + i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] + \right. \\
& 2 \text{ArcCosh}[cx] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + \\
& 2 \text{ArcCosh}[cx] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - \\
& \left. 2 \text{PolyLog} \left[ 2, \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 2 \text{PolyLog} \left[ 2, -\frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] \right) - \\
& \frac{1}{32 d^{3/2} e^{3/2}} i \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right] \right)
\end{aligned}$$

Problem 513: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d + e x^2)^3} dx$$

Optimal (type 4, 1234 leaves, 34 steps):

$$\begin{aligned}
 & - \frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{16 (-d)^{3/2} (c^2 d+e) (\sqrt{-d}-\sqrt{e} x)} - \frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{16 (-d)^{3/2} (c^2 d+e) (\sqrt{-d}+\sqrt{e} x)} - \frac{a+b \operatorname{ArcCosh}[c x]}{16 (-d)^{3/2} \sqrt{e} (\sqrt{-d}-\sqrt{e} x)^2} \\
 & + \frac{3 (a+b \operatorname{ArcCosh}[c x])}{16 d^2 \sqrt{e} (\sqrt{-d}-\sqrt{e} x)} + \frac{a+b \operatorname{ArcCosh}[c x]}{16 (-d)^{3/2} \sqrt{e} (\sqrt{-d}+\sqrt{e} x)^2} + \frac{3 (a+b \operatorname{ArcCosh}[c x])}{16 d^2 \sqrt{e} (\sqrt{-d}+\sqrt{e} x)} - \frac{b c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}\right]}{8 d (c \sqrt{-d}-\sqrt{e})^{3/2} (c \sqrt{-d}+\sqrt{e})^{3/2} \sqrt{e}} + \\
 & + \frac{3 b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}\right]}{8 d^2 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{e}} + \frac{b c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}\right]}{8 d (c \sqrt{-d}-\sqrt{e})^{3/2} (c \sqrt{-d}+\sqrt{e})^{3/2} \sqrt{e}} - \frac{3 b c \operatorname{ArcTanh}\left[\frac{\sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{1+c x}}{\sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{1+c x}}\right]}{8 d^2 \sqrt{c \sqrt{-d}-\sqrt{e}} \sqrt{c \sqrt{-d}+\sqrt{e}} \sqrt{e}} + \\
 & + \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} - \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} + \\
 & - \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} - \frac{3 (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} - \frac{3 b \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} + \\
 & - \frac{3 b \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} - \frac{3 b \operatorname{PolyLog}\left[2,-\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}} + \frac{3 b \operatorname{PolyLog}\left[2,\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{16 (-d)^{5/2} \sqrt{e}}
 \end{aligned}$$

Result (type 4, 1593 leaves):

$$\begin{aligned}
 & \frac{a x}{4 d (d+e x^2)^2} + \frac{3 a x}{8 d^2 (d+e x^2)} + \frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 d^{5/2} \sqrt{e}} + \\
 & b \left( \frac{3 \left( \frac{\operatorname{ArcCosh}[c x]}{-i \sqrt{d}+\sqrt{e} x} + \frac{c \operatorname{Log}\left[\frac{2 e^{i \sqrt{e}+c^2 \sqrt{d} x-i \sqrt{-c^2 d-e}} \sqrt{1+c x} \sqrt{1+c x}}{c \sqrt{-c^2 d-e} (\sqrt{d}+i \sqrt{e} x)}\right]}{\sqrt{-c^2 d-e}} \right)}{16 d^2 \sqrt{e}} - \frac{3 \left( \frac{\operatorname{ArcCosh}[c x]}{i \sqrt{d}+\sqrt{e} x} - \frac{c \operatorname{Log}\left[\frac{2 e^{-i \sqrt{e}-i c^2 \sqrt{d} x+\sqrt{-c^2 d-e}} \sqrt{1+c x} \sqrt{1+c x}}{c \sqrt{-c^2 d-e} (i \sqrt{d}+\sqrt{e} x)}\right]}{\sqrt{-c^2 d-e}} \right)}{16 d^2 \sqrt{e}} \right) + \frac{1}{16 d^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
& \left( \frac{i \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} - c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx}}{c^3 (d+i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right)}{16 d^{3/2}} \right. \\
& \left. + \frac{i \left( \frac{c \sqrt{-1+cx} \sqrt{1+cx}}{(c^2 d+e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcCosh}[cx]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} - \frac{c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log} \left[ \frac{e \sqrt{c^2 d+e} (-i \sqrt{e} + c^2 \sqrt{d} x + \sqrt{c^2 d+e} \sqrt{-1+cx} \sqrt{1+cx}}{c^3 (d-i \sqrt{d} \sqrt{e} x)} \right] \right)}{\sqrt{e} (c^2 d+e)^{3/2}} \right)}{16 d^{3/2}} \right) + \\
& \frac{1}{32 d^{5/2} \sqrt{e}} \left( 3 i \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} + i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] \right) + \right. \\
& 2 \text{ArcCosh}[cx] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 - \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + \\
& 2 \text{ArcCosh}[cx] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] + 4 i \text{ArcSin} \left[ \frac{\sqrt{1 + \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{Log} \left[ 1 + \frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - \\
& \left. 2 \text{PolyLog} \left[ 2, \frac{i (-c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] - 2 \text{PolyLog} \left[ 2, -\frac{i (c \sqrt{d} + \sqrt{c^2 d+e}) e^{-\text{ArcCosh}[cx]}}{\sqrt{e}} \right] \right) - \\
& \frac{1}{32 d^{5/2} \sqrt{e}} \left( 3 i \left( \text{ArcCosh}[cx]^2 + 8 i \text{ArcSin} \left[ \frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ArcTanh} \left[ \frac{(c \sqrt{d} - i \sqrt{e}) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[cx] \right]}{\sqrt{c^2 d+e}} \right] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + \\
& 2 \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - \\
& \left. \left. \left. 2 \operatorname{PolyLog}\left[2, -\frac{i(-c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i(c\sqrt{d} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)\right)
\end{aligned}$$

**Problem 519:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (f x)^m (d + e x^2)^3 (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 558 leaves, 8 steps):

$$\begin{aligned}
& \left( b e \left( 3 c^2 d e (7 + m)^2 (12 + 7 m + m^2) + 3 c^4 d^2 (35 + 12 m + m^2)^2 + e^2 (360 + 342 m + 119 m^2 + 18 m^3 + m^4) \right) (f x)^{2+m} (1 - c^2 x^2) \right) / \\
& \left( c^5 f^2 (3 + m)^2 (5 + m)^2 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x} \right) + \frac{b e^2 \left( 3 c^2 d (7 + m)^2 + e (30 + 11 m + m^2) \right) (f x)^{4+m} (1 - c^2 x^2)}{c^3 f^4 (5 + m)^2 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b e^3 (f x)^{6+m} (1 - c^2 x^2)}{c f^6 (7 + m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{d^3 (f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{f (1 + m)} + \frac{3 d^2 e (f x)^{3+m} (a + b \operatorname{ArcCosh}[c x])}{f^3 (3 + m)} + \\
& \frac{3 d e^2 (f x)^{5+m} (a + b \operatorname{ArcCosh}[c x])}{f^5 (5 + m)} + \frac{e^3 (f x)^{7+m} (a + b \operatorname{ArcCosh}[c x])}{f^7 (7 + m)} - \left( b \left( \frac{c^6 d^3 (3 + m) (5 + m) (7 + m)}{1 + m} + \frac{1}{(3 + m) (5 + m) (7 + m)} \right) \right) \\
& e (2 + m) \left( 3 c^2 d e (7 + m)^2 (12 + 7 m + m^2) + 3 c^4 d^2 (35 + 12 m + m^2)^2 + e^2 (360 + 342 m + 119 m^2 + 18 m^3 + m^4) \right) (f x)^{2+m} \\
& \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2 + m}{2}, \frac{4 + m}{2}, c^2 x^2\right] / \left( c^5 f^2 (2 + m) (3 + m) (5 + m) (7 + m) \sqrt{-1 + c x} \sqrt{1 + c x} \right)
\end{aligned}$$



Result (type 6, 3434 leaves):

$$\begin{aligned}
& \frac{a d^3 x (f x)^m}{1+m} + \frac{3 a d^2 e x^3 (f x)^m}{3+m} + \frac{3 a d e^2 x^5 (f x)^m}{5+m} + \frac{a e^3 x^7 (f x)^m}{7+m} + \frac{1}{c} b d^3 (c x)^{-m} (f x)^m \\
& \left( -\frac{1}{1+m} 12 (c x)^m \left( \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) - \right. \\
& \quad \left. \left( \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \right. \right. \\
& \quad \left. \left. \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \frac{(c x)^{1+m} \operatorname{ArcCosh}[c x]}{1+m} \Bigg) + \\
& \frac{1}{c} 3 b d^2 e x^2 (c x)^{-2-m} (f x)^m \left( -\frac{1}{3+m} 4 (c x)^m \left( \left( 3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \right. \right. \\
& \quad \left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \quad \left( 3 \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Bigg) + \\
& (-1+c x)^{3/2} \sqrt{1+c x} \left( \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. 3 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \\
& \quad \left( 7 (-1+c x) \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. \left. 5 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{(c x)^{3+m} \text{ArcCosh}[c x]}{3+m} \right) + \frac{1}{c} 3 b d e^2 x^4 (c x)^{-4-m} (f x)^m \left( -\frac{1}{5+m} \left( \left( 12 (c x)^m \sqrt{-1+c x} \sqrt{1+c x} \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \left( 6 \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \right. \right. \\
& \left. \left. (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \text{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) \right) \right) - \\
& \left( 12 (c x)^m \sqrt{\frac{-1+c x}{1+c x}} \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \left( 6 \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \right. \\
& \left. 4 m (-1+c x) \text{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] - (-1+c x) \text{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) + \\
& \left( 40 (c x)^m (-1+c x)^{3/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \left( 30 \text{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \right. \\
& \left. 3 (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \text{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) \right) + \\
& \left( 112 (c x)^m (-1+c x)^{5/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \left( 70 \text{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \right. \\
& \left. 5 (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \text{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) \right) + \\
& \left( 108 (c x)^m (-1+c x)^{7/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \\
& \left( 7 \left( 18 \text{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{9}{2}, 1-m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \text{AppellF1}\left[\frac{9}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -m, \frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) \right) \right) + \left( 44 (c x)^m (-1+c x)^{9/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \\
& \left( 9 \left( 22 \text{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{11}{2}, 1-m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{11}{2}, -m, \frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) \right) \right) \right) + \frac{(c x)^{5+m} \text{ArcCosh}[c x]}{5+m} \Bigg) + \\
& \frac{1}{c} b e^3 x^6 (c x)^{-6-m} (f x)^m \left( -\frac{1}{7+m} \left( \left( 12 (c x)^m \sqrt{-1+c x} \sqrt{1+c x} \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) / \right. \right. \\
& \left. \left( 6 \text{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \right. \right. \\
& \left. \left. (-1+c x) \left( 4 m \text{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] + \text{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x) \right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 12 (c x)^m \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. 4 m (-1+c x) \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - (-1+c x) \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \\
& \left( 60 (c x)^m (-1+c x)^{3/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. 3 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) + \\
& \left( 252 (c x)^m (-1+c x)^{5/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. 5 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) + \\
& \left( 468 (c x)^m (-1+c x)^{7/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 7 \left( 18 \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{9}{2}, 1-m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{9}{2}, -m, \frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \left( 484 (c x)^m (-1+c x)^{9/2} \sqrt{1+c x} \right. \\
& \quad \left. \operatorname{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 9 \left( 22 \operatorname{AppellF1}\left[\frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{11}{2}, 1-m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{11}{2}, -m, \frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \\
& \left( 260 (c x)^m (-1+c x)^{11/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{11}{2}, -m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 11 \left( 26 \operatorname{AppellF1}\left[\frac{11}{2}, -m, -\frac{1}{2}, \frac{13}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{13}{2}, 1-m, -\frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{13}{2}, -m, \frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \\
& \left( 60 (c x)^m (-1+c x)^{13/2} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{13}{2}, -m, -\frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \\
& \left( 13 \left( 30 \operatorname{AppellF1}\left[\frac{13}{2}, -m, -\frac{1}{2}, \frac{15}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{15}{2}, 1-m, -\frac{1}{2}, \frac{17}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{15}{2}, -m, \frac{1}{2}, \frac{17}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \frac{(c x)^{7+m} \operatorname{ArcCosh}[c x]}{7+m}
\end{aligned}$$

Problem 520: Result unnecessarily involves higher level functions and more than twice size of optimal

## antiderivative.

$$\int (f x)^m (d + e x^2)^2 (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 353 leaves, 7 steps):

$$\frac{b e (2 c^2 d (5+m)^2 + e (12+7m+m^2)) (f x)^{2+m} (1-c^2 x^2)}{c^3 f^2 (3+m)^2 (5+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b e^2 (f x)^{4+m} (1-c^2 x^2)}{c f^4 (5+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} +$$

$$\frac{d^2 (f x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{f (1+m)} + \frac{2 d e (f x)^{3+m} (a + b \operatorname{ArcCosh}[c x])}{f^3 (3+m)} + \frac{e^2 (f x)^{5+m} (a + b \operatorname{ArcCosh}[c x])}{f^5 (5+m)} -$$

$$\left( b \left( \frac{c^4 d^2 (3+m) (5+m)}{1+m} + \frac{e (2+m) (2 c^2 d (5+m)^2 + e (12+7m+m^2))}{(3+m) (5+m)} \right) (f x)^{2+m} \sqrt{1-c^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) /$$

$$(c^3 f^2 (2+m) (3+m) (5+m) \sqrt{-1+c x} \sqrt{1+c x})$$

Result (type 6, 2079 leaves):

$$\frac{a d^2 x (f x)^m}{1+m} + \frac{2 a d e x^3 (f x)^m}{3+m} + \frac{a e^2 x^5 (f x)^m}{5+m} + \frac{1}{c} b d^2 (c x)^{-m} (f x)^m$$

$$\left( -\frac{1}{1+m} 12 (c x)^m \left( \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) + \right.$$

$$\left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) \right) -$$

$$\left( \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x)\right] + (-1+c x) \right.$$

$$\left. \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) \right) + \frac{(c x)^{1+m} \operatorname{ArcCosh}[c x]}{1+m} \Bigg) +$$

$$\frac{1}{c} 2 b d e x^2 (c x)^{-2-m} (f x)^m \left( -\frac{1}{3+m} 4 (c x)^m \left( 3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) / \right.$$

$$\left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2} (1-c x)\right] + \right.$$

$$\left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2} (1-c x)\right] \right) \right) -$$

$$\begin{aligned}
& \left( 3 \sqrt{\frac{-1+cx}{1+cx}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) + \\
& (-1+cx)^{3/2} \sqrt{1+cx} \left( \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \right. \\
& \quad \left. \left. 3(-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \right) + \\
& \left( 7(-1+cx) \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 5(-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \Big) + \\
& \left. \frac{(cx)^{3+m} \operatorname{ArcCosh}[cx]}{3+m} \right) + \frac{1}{c} b e^2 x^4 (cx)^{-4-m} (fx)^m \left( -\frac{1}{5+m} \left( \left( 12 (cx)^m \sqrt{-1+cx} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \right. \right. \\
& \quad \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. \left. (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) \right) - \\
& \left( 12 (cx)^m \sqrt{\frac{-1+cx}{1+cx}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 4m(-1+cx) \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] - (-1+cx) \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) + \\
& \left( 40 (cx)^m (-1+cx)^{3/2} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 3(-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) + \\
& \left( 112 (cx)^m (-1+cx)^{5/2} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \right. \\
& \quad \left. 5(-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) \right) + \\
& \left( 108 (cx)^m (-1+cx)^{7/2} \sqrt{1+cx} \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] \right) / \\
& \quad \left( 7 \left( 18 \operatorname{AppellF1}\left[\frac{7}{2}, -m, -\frac{1}{2}, \frac{9}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + (-1+cx) \left( 4m \operatorname{AppellF1}\left[\frac{9}{2}, 1-m, -\frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx)\right] + \operatorname{AppellF1}\left[\frac{9}{2}, \right. \right. \right. \right.
\end{aligned}$$

$$\left( 9 \left( 22 \operatorname{AppellF1} \left[ \frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx) \right] + (-1+cx) \left( 4m \operatorname{AppellF1} \left[ \frac{11}{2}, 1-m, -\frac{1}{2}, \frac{13}{2}, 1-cx, \frac{1}{2}(1-cx) \right] + \operatorname{AppellF1} \left[ \frac{11}{2}, -m, \frac{1}{2}, \frac{13}{2}, 1-cx, \frac{1}{2}(1-cx) \right] \right) \right) + \frac{(cx)^{5+m} \operatorname{ArcCosh}[cx]}{5+m} \right) / \left( 44 (cx)^m (-1+cx)^{9/2} \sqrt{1+cx} \operatorname{AppellF1} \left[ \frac{9}{2}, -m, -\frac{1}{2}, \frac{11}{2}, 1-cx, \frac{1}{2}(1-cx) \right] \right)$$

**Problem 521: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (fx)^m (d+ex^2) (a+b \operatorname{ArcCosh}[cx]) dx$$

Optimal (type 5, 198 leaves, 5 steps):

$$\frac{be (fx)^{2+m} \sqrt{-1+cx} \sqrt{1+cx}}{cf^2 (3+m)^2} + \frac{d (fx)^{1+m} (a+b \operatorname{ArcCosh}[cx])}{f (1+m)} + \frac{e (fx)^{3+m} (a+b \operatorname{ArcCosh}[cx])}{f^3 (3+m)} - \frac{b (e (1+m) (2+m) + c^2 d (3+m)^2) (fx)^{2+m} \sqrt{1-c^2 x^2} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2 \right]}{cf^2 (1+m) (2+m) (3+m)^2 \sqrt{-1+cx} \sqrt{1+cx}}$$

Result (type 6, 1044 leaves):

$$\begin{aligned}
& \frac{a d x (f x)^m}{1+m} + \frac{a e x^3 (f x)^m}{3+m} + \frac{1}{c} b d (c x)^{-m} (f x)^m \\
& \left( -\frac{1}{1+m} 12 (c x)^m \left( \left( \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \quad \left( \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + (-1+c x) \right. \\
& \quad \left. \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \frac{(c x)^{1+m} \operatorname{ArcCosh}[c x]}{1+m} \Bigg) + \\
& \frac{1}{c} b e x^2 (c x)^{-2-m} (f x)^m \left( -\frac{1}{3+m} 4 (c x)^m \left( \left( 3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \right. \right. \\
& \quad \left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) - \\
& \quad \left( 3 \sqrt{\frac{-1+c x}{1+c x}} \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] - \operatorname{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Bigg) + \\
& \quad (-1+c x)^{3/2} \sqrt{1+c x} \left( \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -m, -\frac{1}{2}, \frac{5}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. 3 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, 1-m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \operatorname{AppellF1}\left[\frac{5}{2}, -m, \frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \right) + \\
& \quad \left( 7 (-1+c x) \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) / \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -m, -\frac{1}{2}, \frac{7}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \\
& \quad \left. 5 (-1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, 1-m, -\frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] + \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{7}{2}, -m, \frac{1}{2}, \frac{9}{2}, 1-c x, \frac{1}{2}(1-c x)\right] \right) \right) \Bigg) + \frac{(c x)^{3+m} \operatorname{ArcCosh}[c x]}{3+m} \Bigg)
\end{aligned}$$

### Problem 529: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{d + e x^2} dx$$

Optimal (type 4, 763 leaves, 22 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{ArcCosh}[c x])^2 \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{ArcCosh}[c x])^2 \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} + \\ & \frac{(a + b \operatorname{ArcCosh}[c x])^2 \operatorname{Log}\left[1 - \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{ArcCosh}[c x])^2 \operatorname{Log}\left[1 + \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{2 \sqrt{-d} \sqrt{e}} - \\ & \frac{b (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} + \frac{b (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} - \\ & \frac{b (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} + \frac{b (a + b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} + \\ & \frac{b^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} - \frac{b^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} + \frac{b^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} - \frac{b^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{e} e^{\operatorname{ArcCosh}[c x]}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right]}{\sqrt{-d} \sqrt{e}} \end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{d + e x^2} dx$$

### Problem 546: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(d + e x^2) (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{1}{(d + e x^2) (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):



???

**Problem 547: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(d + e x^2)^2 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(d + e x^2)^2 (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 550: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(d + e x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(d + e x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 551: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(d + e x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(d + e x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

## Test results for the 296 problems in "7.2.5 Inverse hyperbolic cosine functions.m"

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCosh}[c x]}{d + e x} dx$$

Optimal (type 4, 178 leaves, 8 steps):

$$-\frac{\text{ArcCosh}[c x]^2}{2 e} + \frac{\text{ArcCosh}[c x] \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{\text{ArcCosh}[c x] \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{\text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{\text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e}$$

Result (type 4, 281 leaves):

$$\frac{1}{e} \left( \frac{1}{2} \text{ArcCosh}[c x]^2 + 4 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(c d - e) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d^2 - e^2}}\right] + \left( \text{ArcCosh}[c x] - 2 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right) \text{Log}\left[1 + \frac{(c d - \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] + \left( \text{ArcCosh}[c x] + 2 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right) \text{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - \text{PolyLog}\left[2, \frac{(-c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - \text{PolyLog}\left[2, -\frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] \right)$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCosh}[c x]}{(d + e x)^4} dx$$

Optimal (type 3, 195 leaves, 6 steps):

$$-\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6(c^2d^2-e^2)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2(cd-e)^2(cd+e)^2(d+ex)} - \frac{\text{ArcCosh}[cx]}{3e(d+ex)^3} + \frac{c^3(2c^2d^2+e^2)\text{ArcTanh}\left[\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right]}{3(cd-e)^{5/2}e(cd+e)^{5/2}}$$

Result (type 3, 244 leaves):

$$\frac{1}{6} \left( \frac{c\sqrt{-1+cx}\sqrt{1+cx}(e^2-c^2d(4d+3ex))}{(-c^2d^2+e^2)^2(d+ex)^2} - \frac{2\text{ArcCosh}[cx]}{e(d+ex)^3} - \frac{i c^3(2c^2d^2+e^2)\text{Log}\left[\frac{12e^2(-cd+e)^2(cd+e)^2(-ie-i c^2dx+\sqrt{-c^2d^2+e^2}\sqrt{-1+cx}\sqrt{1+cx})}{c^3\sqrt{-c^2d^2+e^2}(2c^2d^2+e^2)(d+ex)}\right]}{e(-cd+e)^2(cd+e)^2\sqrt{-c^2d^2+e^2}} \right)$$

**Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcCosh}[cx]^2}{d+ex} dx$$

Optimal (type 4, 272 leaves, 10 steps):

$$-\frac{\text{ArcCosh}[cx]^3}{3e} + \frac{\text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[cx]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{\text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[cx]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{2 \text{ArcCosh}[cx] \text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[cx]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} +$$

$$\frac{2 \text{ArcCosh}[cx] \text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[cx]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e} - \frac{2 \text{PolyLog}\left[3, -\frac{e e^{\text{ArcCosh}[cx]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} - \frac{2 \text{PolyLog}\left[3, -\frac{e e^{\text{ArcCosh}[cx]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e}$$

Result (type 4, 766 leaves):

$$\begin{aligned}
& -\frac{1}{3e} \left( \text{ArcCosh}[cx]^3 - 3 \text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{(cd - \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[cx]}}{e}\right] + \right. \\
& 12i \text{ArcCosh}[cx] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{cd}{e}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(cd - \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[cx]}}{e}\right] - 3 \text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{(cd + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[cx]}}{e}\right] - \\
& 12i \text{ArcCosh}[cx] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{cd}{e}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(cd + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[cx]}}{e}\right] - 3 \text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[cx]}}{cd - \sqrt{c^2 d^2 - e^2}}\right] - \\
& 3 \text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[cx]}}{cd + \sqrt{c^2 d^2 - e^2}}\right] + 3 \text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{(cd + \sqrt{c^2 d^2 - e^2}) \left(cx - \sqrt{\frac{-1+cx}{1+cx}} (1+cx)\right)}{e}\right] + \\
& 12i \text{ArcCosh}[cx] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{cd}{e}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(cd + \sqrt{c^2 d^2 - e^2}) \left(cx - \sqrt{\frac{-1+cx}{1+cx}} (1+cx)\right)}{e}\right] + \\
& 3 \text{ArcCosh}[cx]^2 \text{Log}\left[1 + \frac{(-cd + \sqrt{c^2 d^2 - e^2}) \left(-cx + \sqrt{\frac{-1+cx}{1+cx}} (1+cx)\right)}{e}\right] - 12i \text{ArcCosh}[cx] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{cd}{e}}}{\sqrt{2}}\right] \\
& \text{Log}\left[1 + \frac{(-cd + \sqrt{c^2 d^2 - e^2}) \left(-cx + \sqrt{\frac{-1+cx}{1+cx}} (1+cx)\right)}{e}\right] - 6 \text{ArcCosh}[cx] \text{PolyLog}\left[2, \frac{e e^{\text{ArcCosh}[cx]}}{-cd + \sqrt{c^2 d^2 - e^2}}\right] - \\
& \left. 6 \text{ArcCosh}[cx] \text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[cx]}}{cd + \sqrt{c^2 d^2 - e^2}}\right] + 6 \text{PolyLog}\left[3, \frac{e e^{\text{ArcCosh}[cx]}}{-cd + \sqrt{c^2 d^2 - e^2}}\right] + 6 \text{PolyLog}\left[3, -\frac{e e^{\text{ArcCosh}[cx]}}{cd + \sqrt{c^2 d^2 - e^2}}\right] \right)
\end{aligned}$$

**Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcCosh}[c x]^2}{(d + e x)^2} dx$$

Optimal (type 4, 259 leaves, 10 steps):

$$\begin{aligned} & -\frac{\text{ArcCosh}[c x]^2}{e (d + e x)} + \frac{2 c \text{ArcCosh}[c x] \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} - \\ & \frac{2 c \text{ArcCosh}[c x] \text{Log}\left[1 + \frac{e e^{\text{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} + \frac{2 c \text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} - \frac{2 c \text{PolyLog}\left[2, -\frac{e e^{\text{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} \end{aligned}$$

Result (type 4, 848 leaves):

$$\begin{aligned}
& -\frac{1}{e} c \left( \frac{\text{ArcCosh}[c x]^2}{c d + c e x} + \right. \\
& \frac{1}{\sqrt{-c^2 d^2 + e^2}} 2 \left( 2 \text{ArcCosh}[c x] \text{ArcTan}\left[\frac{(c d + e) \text{Coth}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] - 2 i \text{ArcCos}\left[-\frac{c d}{e}\right] \text{ArcTan}\left[\frac{(-c d + e) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) + \\
& \left. \left( \text{ArcCos}\left[-\frac{c d}{e}\right] + 2 \left( \text{ArcTan}\left[\frac{(c d + e) \text{Coth}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] + \text{ArcTan}\left[\frac{(-c d + e) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \right) \right) \\
& \text{Log}\left[\frac{\sqrt{-c^2 d^2 + e^2} e^{-\frac{1}{2} \text{ArcCosh}[c x]}}{\sqrt{2} \sqrt{e} \sqrt{c d + c e x}}\right] + \left( \text{ArcCos}\left[-\frac{c d}{e}\right] - \right. \\
& \left. 2 \left( \text{ArcTan}\left[\frac{(c d + e) \text{Coth}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] + \text{ArcTan}\left[\frac{(-c d + e) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{-c^2 d^2 + e^2} e^{\frac{1}{2} \text{ArcCosh}[c x]}}{\sqrt{2} \sqrt{e} \sqrt{c d + c e x}}\right] - \\
& \left( \text{ArcCos}\left[-\frac{c d}{e}\right] + 2 \text{ArcTan}\left[\frac{(-c d + e) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \text{Log}\left[\frac{(c d + e) (c d - e + i \sqrt{-c^2 d^2 + e^2}) (-1 + \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{e (c d + e + i \sqrt{-c^2 d^2 + e^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] - \\
& \left( \text{ArcCos}\left[-\frac{c d}{e}\right] - 2 \text{ArcTan}\left[\frac{(-c d + e) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \text{Log}\left[\frac{(c d + e) (-c d + e + i \sqrt{-c^2 d^2 + e^2}) (1 + \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{e (c d + e + i \sqrt{-c^2 d^2 + e^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] + \\
& i \left( \text{PolyLog}\left[2, \frac{(c d - i \sqrt{-c^2 d^2 + e^2}) (c d + e - i \sqrt{-c^2 d^2 + e^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{e (c d + e + i \sqrt{-c^2 d^2 + e^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] \right) - \\
& \left. \text{PolyLog}\left[2, \frac{(c d + i \sqrt{-c^2 d^2 + e^2}) (c d + e - i \sqrt{-c^2 d^2 + e^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{e (c d + e + i \sqrt{-c^2 d^2 + e^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] \right) \right) \right)
\end{aligned}$$

**Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcCosh}[c x]^2}{(d + e x)^3} dx$$

Optimal (type 4, 352 leaves, 13 steps):

$$\begin{aligned}
& - \frac{c \sqrt{-\frac{1-cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx]}{(c^2 d^2 - e^2) (d+ex)} - \frac{\operatorname{ArcCosh}[cx]^2}{2e (d+ex)^2} + \frac{c^3 d \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[cx]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} - \\
& \frac{c^3 d \operatorname{ArcCosh}[cx] \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[cx]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} + \frac{c^2 \operatorname{Log}[d+ex]}{e (c^2 d^2 - e^2)} + \frac{c^3 d \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[cx]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} - \frac{c^3 d \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[cx]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}}
\end{aligned}$$

Result (type 4, 936 leaves):

$$\begin{aligned}
& c^2 \left( -\frac{\sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx]}{(cd-e)(cd+e)(cd+cx)} - \frac{\operatorname{ArcCosh}[cx]^2}{2e(cd+cx)^2} + \frac{\operatorname{Log}\left[1+\frac{ex}{d}\right]}{c^2 d^2 e - e^3} + \right. \\
& \frac{1}{e(-c^2 d^2 + e^2)^{3/2}} cd \left( 2 \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\frac{(cd+e) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] - 2i \operatorname{ArcCos}\left[-\frac{cd}{e}\right] \operatorname{ArcTan}\left[\frac{(-cd+e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) + \\
& \left( \operatorname{ArcCos}\left[-\frac{cd}{e}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(cd+e) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cd+e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-c^2 d^2 + e^2} e^{-\frac{1}{2} \operatorname{ArcCosh}[cx]}}{\sqrt{2} \sqrt{e} \sqrt{cd+cx}}\right] + \left( \operatorname{ArcCos}\left[-\frac{cd}{e}\right] - \right. \\
& \left. 2 \left( \operatorname{ArcTan}\left[\frac{(cd+e) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cd+e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-c^2 d^2 + e^2} e^{\frac{1}{2} \operatorname{ArcCosh}[cx]}}{\sqrt{2} \sqrt{e} \sqrt{cd+cx}}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{cd}{e}\right] + 2 \operatorname{ArcTan}\left[\frac{(-cd+e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \operatorname{Log}\left[\frac{(cd+e)(cd-e+i\sqrt{-c^2 d^2 + e^2})(-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2 d^2 + e^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{cd}{e}\right] - 2 \operatorname{ArcTan}\left[\frac{(-cd+e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] \right) \operatorname{Log}\left[\frac{(cd+e)(-cd+e+i\sqrt{-c^2 d^2 + e^2})(1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2 d^2 + e^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(cd-i\sqrt{-c^2 d^2 + e^2})(cd+e-i\sqrt{-c^2 d^2 + e^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2 d^2 + e^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(cd+i\sqrt{-c^2 d^2 + e^2})(cd+e-i\sqrt{-c^2 d^2 + e^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2 d^2 + e^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] \right) \right)
\end{aligned}$$

**Problem 17: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[cx]}{d + ex} dx$$

Optimal (type 4, 195 leaves, 8 steps):



$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcCosh}[c x])^2}{2 b e} + \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} + \\
& \frac{(a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e}
\end{aligned}$$

Result (type 4, 294 leaves):

$$\begin{aligned}
& \frac{a \operatorname{Log}[d + e x]}{e} + \\
& \frac{1}{e} b \left( \frac{1}{2} \operatorname{ArcCosh}[c x]^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(c d - e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{c^2 d^2 - e^2}}\right] + \left( \operatorname{ArcCosh}[c x] - 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[ \right. \right. \\
& \left. \left. 1 + \frac{(c d - \sqrt{c^2 d^2 - e^2}) e^{-\operatorname{ArcCosh}[c x]}}{e} \right] + \left( \operatorname{ArcCosh}[c x] + 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\operatorname{ArcCosh}[c x]}}{e} \right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(-c d + \sqrt{c^2 d^2 - e^2}) e^{-\operatorname{ArcCosh}[c x]}}{e}\right] - \operatorname{PolyLog}\left[2, -\frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\operatorname{ArcCosh}[c x]}}{e}\right] \right)
\end{aligned}$$

**Problem 20: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d + e x)^4} dx$$

Optimal (type 3, 202 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x}}{6 (c^2 d^2 - e^2) (d + e x)^2} - \frac{b c^3 d \sqrt{-1 + c x} \sqrt{1 + c x}}{2 (c d - e)^2 (c d + e)^2 (d + e x)} - \frac{a + b \operatorname{ArcCosh}[c x]}{3 e (d + e x)^3} + \frac{b c^3 (2 c^2 d^2 + e^2) \operatorname{ArcTanh}\left[\frac{\sqrt{c d + e} \sqrt{1 + c x}}{\sqrt{c d - e} \sqrt{-1 + c x}}\right]}{3 (c d - e)^{5/2} e (c d + e)^{5/2}}
\end{aligned}$$

Result (type 3, 259 leaves):

$$-\frac{1}{6e} \left( \frac{2a + \frac{bce\sqrt{-1+cx}\sqrt{1+cx}(d+ex)(-e^2+c^2d(4d+3ex))}{(-c^2d^2+e^2)^2}}{(d+ex)^3} + \frac{2b \operatorname{ArcCosh}[cx]}{(d+ex)^3} + \frac{i b c^3 (2c^2d^2 + e^2) \operatorname{Log}\left[\frac{12e^2(-cd+e)^2(cd+e)^2(-ie-i c^2dx + \sqrt{-c^2d^2+e^2}\sqrt{-1+cx}\sqrt{1+cx})}{b c^3 \sqrt{-c^2d^2+e^2} (2c^2d^2+e^2)(d+ex)}\right]}{(-cd+e)^2(cd+e)^2\sqrt{-c^2d^2+e^2}} \right)$$

**Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCosh}[cx])^2}{d + ex} dx$$

Optimal (type 4, 303 leaves, 10 steps):

$$-\frac{(a + b \operatorname{ArcCosh}[cx])^3}{3be} + \frac{(a + b \operatorname{ArcCosh}[cx])^2 \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[cx]}}{cd - \sqrt{c^2d^2 - e^2}}\right]}{e} + \frac{(a + b \operatorname{ArcCosh}[cx])^2 \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[cx]}}{cd + \sqrt{c^2d^2 - e^2}}\right]}{e} + \frac{2b(a + b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[cx]}}{cd - \sqrt{c^2d^2 - e^2}}\right]}{e} + \frac{2b(a + b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[cx]}}{cd + \sqrt{c^2d^2 - e^2}}\right]}{e} - \frac{2b^2 \operatorname{PolyLog}\left[3, -\frac{e e^{\operatorname{ArcCosh}[cx]}}{cd - \sqrt{c^2d^2 - e^2}}\right]}{e} - \frac{2b^2 \operatorname{PolyLog}\left[3, -\frac{e e^{\operatorname{ArcCosh}[cx]}}{cd + \sqrt{c^2d^2 - e^2}}\right]}{e}$$

Result (type 4, 1064 leaves):

$$\frac{1}{3e} \left( 3a^2 \operatorname{Log}[d + ex] + 6ab \left( \frac{1}{2} \operatorname{ArcCosh}[cx]^2 + 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{cd}{e}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(cd - e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{c^2d^2 - e^2}}\right] + \left( \operatorname{ArcCosh}[cx] - 2i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{cd}{e}}}{\sqrt{2}}\right] \right) \right) \right)$$

$$\begin{aligned}
& \text{Log}\left[1 + \frac{(c d - \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] + \left( \text{ArcCosh}[c x] + 2 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right) \text{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - \\
& \text{PolyLog}\left[2, \frac{(-c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - \text{PolyLog}\left[2, -\frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - \\
& b^2 \left( \text{ArcCosh}[c x]^3 - 3 \text{ArcCosh}[c x]^2 \text{Log}\left[1 + \frac{(c d - \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] + 12 i \text{ArcCosh}[c x] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right. \\
& \left. \text{Log}\left[1 + \frac{(c d - \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - 3 \text{ArcCosh}[c x]^2 \text{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - \right. \\
& \left. 12 i \text{ArcCosh}[c x] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) e^{-\text{ArcCosh}[c x]}}{e}\right] - 3 \text{ArcCosh}[c x]^2 \text{Log}\left[1 + \frac{e^{\text{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right] - \right. \\
& \left. 3 \text{ArcCosh}[c x]^2 \text{Log}\left[1 + \frac{e^{\text{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right] + 3 \text{ArcCosh}[c x]^2 \text{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) \left(c x - \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)}{e}\right] + \right. \\
& \left. 12 i \text{ArcCosh}[c x] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(c d + \sqrt{c^2 d^2 - e^2}) \left(c x - \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)}{e}\right] + \right. \\
& \left. 3 \text{ArcCosh}[c x]^2 \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 - e^2}) \left(-c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)}{e}\right] - 12 i \text{ArcCosh}[c x] \text{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \right. \\
& \left. \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 - e^2}) \left(-c x + \sqrt{\frac{-1+c x}{1+c x}} (1+c x)\right)}{e}\right] - 6 \text{ArcCosh}[c x] \text{PolyLog}\left[2, \frac{e^{\text{ArcCosh}[c x]}}{-c d + \sqrt{c^2 d^2 - e^2}}\right] - \right.
\end{aligned}$$

$$\left. \left. \left. 6 \operatorname{ArcCosh}[c x] \operatorname{PolyLog}\left[2, -\frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{c d + \sqrt{c^2 d^2 - e^2}}\right] + 6 \operatorname{PolyLog}\left[3, \frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{-c d + \sqrt{c^2 d^2 - e^2}}\right] + 6 \operatorname{PolyLog}\left[3, -\frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{c d + \sqrt{c^2 d^2 - e^2}}\right] \right) \right) \right)$$

**Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{(d + e x)^2} dx$$

Optimal (type 4, 279 leaves, 10 steps):

$$\begin{aligned} & -\frac{(a + b \operatorname{ArcCosh}[c x])^2}{e (d + e x)} + \frac{2 b c (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} - \\ & \frac{2 b c (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} + \frac{2 b^2 c \operatorname{PolyLog}\left[2, -\frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} - \frac{2 b^2 c \operatorname{PolyLog}\left[2, -\frac{e^{e^{\operatorname{ArcCosh}[c x]}}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e \sqrt{c^2 d^2 - e^2}} \end{aligned}$$

Result (type 4, 943 leaves):

$$\begin{aligned}
& -\frac{1}{e} \left( \frac{a^2}{d+ex} - 2abc \left( -\frac{\text{ArcCosh}[cx]}{cd+cx} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{-cd+e}\sqrt{\frac{-1+cx}{1+cx}}}{\sqrt{cd+e}}\right]}{\sqrt{-cd+e}\sqrt{cd+e}} \right) \right) + \\
& b^2c \left( \frac{\text{ArcCosh}[cx]^2}{cd+cx} + \frac{1}{\sqrt{-c^2d^2+e^2}} 2 \left( 2 \text{ArcCosh}[cx] \text{ArcTan}\left[\frac{(cd+e)\text{Coth}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] - \right. \right. \\
& \quad \left. \left. 2i \text{ArcCos}\left[-\frac{cd}{e}\right] \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] + \left( \text{ArcCos}\left[-\frac{cd}{e}\right] + \right. \right. \right. \\
& \quad \left. \left. 2 \left( \text{ArcTan}\left[\frac{(cd+e)\text{Coth}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] + \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \right) \right) \text{Log}\left[\frac{\sqrt{-c^2d^2+e^2} e^{-\frac{1}{2}\text{ArcCosh}[cx]}}{\sqrt{2}\sqrt{e}\sqrt{c(d+ex)}}\right] + \\
& \quad \left( \text{ArcCos}\left[-\frac{cd}{e}\right] - 2 \left( \text{ArcTan}\left[\frac{(cd+e)\text{Coth}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] + \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \right) \\
& \quad \text{Log}\left[\frac{\sqrt{-c^2d^2+e^2} e^{\frac{1}{2}\text{ArcCosh}[cx]}}{\sqrt{2}\sqrt{e}\sqrt{c(d+ex)}}\right] - \left( \text{ArcCos}\left[-\frac{cd}{e}\right] + 2 \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \\
& \quad \text{Log}\left[\frac{(cd+e)(cd-e+i\sqrt{-c^2d^2+e^2})(-1+\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] - \\
& \quad \left( \text{ArcCos}\left[-\frac{cd}{e}\right] - 2 \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \text{Log}\left[\frac{(cd+e)(-cd+e+i\sqrt{-c^2d^2+e^2})(1+\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] + \\
& \quad i \left( \text{PolyLog}\left[2, \frac{(cd-i\sqrt{-c^2d^2+e^2})(cd+e-i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] - \right. \\
& \quad \left. \left. \left. \left. \left. \text{PolyLog}\left[2, \frac{(cd+i\sqrt{-c^2d^2+e^2})(cd+e-i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] \right) \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^2}{(d + e x)^3} dx$$

Optimal (type 4, 380 leaves, 13 steps):

$$\begin{aligned} & - \frac{b c \sqrt{-\frac{1-c x}{1+c x}} (1+c x) (a+b \operatorname{ArcCosh}[c x])}{(c^2 d^2 - e^2) (d+e x)} - \frac{(a+b \operatorname{ArcCosh}[c x])^2}{2 e (d+e x)^2} + \frac{b c^3 d (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} - \\ & \frac{b c^3 d (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e e^{\operatorname{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} + \frac{b^2 c^2 \operatorname{Log}[d+e x]}{e (c^2 d^2 - e^2)} + \frac{b^2 c^3 d \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[c x]}}{c d - \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} - \frac{b^2 c^3 d \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcCosh}[c x]}}{c d + \sqrt{c^2 d^2 - e^2}}\right]}{e (c^2 d^2 - e^2)^{3/2}} \end{aligned}$$

Result (type 4, 1100 leaves):

$$\begin{aligned}
& -\frac{a^2}{2e(d+ex)^2} + 2abc^2 \left( -\frac{\text{ArcCosh}[cx]}{2e(cd+cx)^2} + \frac{e\sqrt{-1+cx}\sqrt{1+cx}}{(cd+e)(cd+e)(cd+cx)} - \frac{2cd \text{ArcTan}\left[\frac{\sqrt{-cd+e}\sqrt{\frac{-1+cx}{1+cx}}}{\sqrt{cd+e}}\right]}{(cd+e)^{3/2}(cd+e)^{3/2}} \right) + \\
& b^2c^2 \left( -\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{ArcCosh}[cx]}{(cd-e)(cd+e)(cd+cx)} - \frac{\text{ArcCosh}[cx]^2}{2e(cd+cx)^2} + \frac{\text{Log}\left[1+\frac{ex}{d}\right]}{c^2d^2e-e^3} + \right. \\
& \frac{1}{e(-c^2d^2+e^2)^{3/2}}cd \left( 2\text{ArcCosh}[cx]\text{ArcTan}\left[\frac{(cd+e)\text{Coth}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] - 2i\text{ArcCos}\left[-\frac{cd}{e}\right]\text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) + \\
& \left( \text{ArcCos}\left[-\frac{cd}{e}\right] + 2\left(\text{ArcTan}\left[\frac{(cd+e)\text{Coth}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] + \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \right) \\
& \text{Log}\left[\frac{\sqrt{-c^2d^2+e^2}e^{-\frac{1}{2}\text{ArcCosh}[cx]}}{\sqrt{2}\sqrt{e}\sqrt{cd+cx}}\right] + \left(\text{ArcCos}\left[-\frac{cd}{e}\right] - \right. \\
& \left. 2\left(\text{ArcTan}\left[\frac{(cd+e)\text{Coth}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] + \text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{-c^2d^2+e^2}e^{\frac{1}{2}\text{ArcCosh}[cx]}}{\sqrt{2}\sqrt{e}\sqrt{cd+cx}}\right] - \\
& \left(\text{ArcCos}\left[-\frac{cd}{e}\right] + 2\text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \text{Log}\left[\frac{(cd+e)(cd-e+i\sqrt{-c^2d^2+e^2})(-1+\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] - \\
& \left(\text{ArcCos}\left[-\frac{cd}{e}\right] - 2\text{ArcTan}\left[\frac{(-cd+e)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right]}{\sqrt{-c^2d^2+e^2}}\right] \right) \text{Log}\left[\frac{(cd+e)(-cd+e+i\sqrt{-c^2d^2+e^2})(1+\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] + \\
& i \left( \text{PolyLog}\left[2, \frac{(cd-i\sqrt{-c^2d^2+e^2})(cd+e-i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{(cd+i\sqrt{-c^2d^2+e^2})(cd+e-i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}{e(cd+e+i\sqrt{-c^2d^2+e^2}\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[cx]\right])}\right] \right) \right)
\end{aligned}$$

### Problem 35: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(d + e x)^2 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(d + e x)^2 (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

### Problem 38: Result more than twice size of optimal antiderivative.

$$\int (d + e x)^m (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 6, 125 leaves, 3 steps):

$$-\frac{\sqrt{2} b (c d + e) \sqrt{-1 + c x} (d + e x)^m \left(\frac{c(d+ex)}{cd+e}\right)^{-m} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1 - m, \frac{3}{2}, \frac{1}{2} (1 - c x), \frac{e(1-cx)}{cd+e}\right]}{c e (1 + m)} + \frac{(d + e x)^{1+m} (a + b \operatorname{ArcCosh}[c x])}{e (1 + m)}$$

Result (type 6, 715 leaves):



$$\begin{aligned}
& \frac{a (d+e x)^{1+m}}{e (1+m)} + \frac{1}{c} b \left( \left( 12 c d (c d+e) \sqrt{\frac{-1+c x}{1+c x}} \left( \frac{c d+e+e(-1+c x)}{c} \right)^m \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1-c x), -\frac{e(-1+c x)}{c d+e} \right] \right) / \right. \\
& \left. \left( e (1+m) \left( -6 (c d+e) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1-c x), -\frac{e(-1+c x)}{c d+e} \right] - 4 e m (-1+c x) \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1-c x), -\frac{e(-1+c x)}{c d+e} \right] + (c d+e) (-1+c x) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1-c x), -\frac{e(-1+c x)}{c d+e} \right] \right) \right) \right) - \\
& \frac{1}{1+m} 12 (c d+e) (d+e x)^m \left( \left( \sqrt{-1+c x} \sqrt{1+c x} \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] \right) / \right. \\
& \left( 6 (c d+e) \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] + 4 e m (-1+c x) \text{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] + \right. \\
& \quad \left. (c d+e) (-1+c x) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] \right) + \left( \sqrt{\frac{-1+c x}{1+c x}} \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] \right) / \\
& \left( -6 (c d+e) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] - 4 e m (-1+c x) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] + \right. \\
& \quad \left. (c d+e) (-1+c x) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} - \frac{c x}{2}, \frac{e-c e x}{c d+e} \right] \right) + \frac{(d+e x)^m (c d+c e x) \text{ArcCosh}[c x]}{e (1+m)} \Bigg)
\end{aligned}$$

**Problem 45: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{ArcCosh}[a x]}{c+d x^2} dx$$

Optimal (type 4, 481 leaves, 18 steps):

$$\begin{aligned}
& \frac{\text{ArcCosh}[a x] \text{Log} \left[ 1 - \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} - \frac{\text{ArcCosh}[a x] \text{Log} \left[ 1 + \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} + \\
& \frac{\text{ArcCosh}[a x] \text{Log} \left[ 1 - \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} - \frac{\text{ArcCosh}[a x] \text{Log} \left[ 1 + \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} - \frac{\text{PolyLog} \left[ 2, -\frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} + \\
& \frac{\text{PolyLog} \left[ 2, \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} - \frac{\text{PolyLog} \left[ 2, -\frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}} + \frac{\text{PolyLog} \left[ 2, \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}} \right]}{2 \sqrt{-c} \sqrt{d}}
\end{aligned}$$

Result (type 4, 791 leaves):

$$\begin{aligned}
 & \frac{1}{2\sqrt{c}\sqrt{d}} \left( 4 \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTanh} \left[ \frac{(a\sqrt{c} - i\sqrt{d}) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[a x] \right]}{\sqrt{a^2 c + d}} \right] - \right. \\
 & 4 \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTanh} \left[ \frac{(a\sqrt{c} + i\sqrt{d}) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh}[a x] \right]}{\sqrt{a^2 c + d}} \right] + \\
 & i \operatorname{ArcCosh}[a x] \operatorname{Log} \left[ 1 - \frac{i(-a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] + 2 \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 - \frac{i(-a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] - \\
 & i \operatorname{ArcCosh}[a x] \operatorname{Log} \left[ 1 + \frac{i(-a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] - 2 \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i(-a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] - \\
 & i \operatorname{ArcCosh}[a x] \operatorname{Log} \left[ 1 - \frac{i(a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] + 2 \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 - \frac{i(a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] + \\
 & i \operatorname{ArcCosh}[a x] \operatorname{Log} \left[ 1 + \frac{i(a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] - 2 \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i(a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] + \\
 & i \operatorname{PolyLog} \left[ 2, -\frac{i(-a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] - i \operatorname{PolyLog} \left[ 2, \frac{i(-a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] - \\
 & \left. i \operatorname{PolyLog} \left[ 2, -\frac{i(a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] + i \operatorname{PolyLog} \left[ 2, \frac{i(a\sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}} \right] \right)
 \end{aligned}$$

**Problem 46:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcCosh}[a x]}{(c + d x^2)^2} dx$$

Optimal (type 4, 774 leaves, 26 steps):

$$\begin{aligned}
 & - \frac{\text{ArcCosh}[a x]}{4 c \sqrt{d} (\sqrt{-c} - \sqrt{d} x)} + \frac{\text{ArcCosh}[a x]}{4 c \sqrt{d} (\sqrt{-c} + \sqrt{d} x)} + \frac{a \text{ArcTanh}\left[\frac{\sqrt{a \sqrt{-c} - \sqrt{d}} \sqrt{1+a x}}{\sqrt{a \sqrt{-c} + \sqrt{d}} \sqrt{-1+a x}}\right]}{2 c \sqrt{a \sqrt{-c} - \sqrt{d}} \sqrt{a \sqrt{-c} + \sqrt{d}} \sqrt{d}} - \\
 & \frac{a \text{ArcTanh}\left[\frac{\sqrt{a \sqrt{-c} + \sqrt{d}} \sqrt{1+a x}}{\sqrt{a \sqrt{-c} - \sqrt{d}} \sqrt{-1+a x}}\right]}{2 c \sqrt{a \sqrt{-c} - \sqrt{d}} \sqrt{a \sqrt{-c} + \sqrt{d}} \sqrt{d}} - \frac{\text{ArcCosh}[a x] \text{Log}\left[1 - \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} + \frac{\text{ArcCosh}[a x] \text{Log}\left[1 + \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} - \\
 & \frac{\text{ArcCosh}[a x] \text{Log}\left[1 - \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} + \frac{\text{ArcCosh}[a x] \text{Log}\left[1 + \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} + \frac{\text{PolyLog}\left[2, -\frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} - \\
 & \frac{\text{PolyLog}\left[2, \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} - \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} + \frac{\text{PolyLog}\left[2, -\frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}} - \frac{\text{PolyLog}\left[2, \frac{\sqrt{d} e^{\text{ArcCosh}[a x]}}{a \sqrt{-c} + \sqrt{-a^2 c - d}}\right]}{4 (-c)^{3/2} \sqrt{d}}
 \end{aligned}$$

Result (type 4, 1080 leaves):

$$\begin{aligned}
& \frac{1}{4 c^{3/2} \sqrt{d}} \left( \frac{\sqrt{c} \operatorname{ArcCosh}[a x]}{-i \sqrt{c} + \sqrt{d} x} + \frac{\sqrt{c} \operatorname{ArcCosh}[a x]}{i \sqrt{c} + \sqrt{d} x} + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(a \sqrt{c} - i \sqrt{d}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[a x]\right]}{\sqrt{a^2 c + d}}\right] \right) - \\
& 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(a \sqrt{c} + i \sqrt{d}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[a x]\right]}{\sqrt{a^2 c + d}}\right] + \\
& i \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 - \frac{i(-a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] + 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(-a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] - \\
& i \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 + \frac{i(-a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] - 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] - \\
& i \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 - \frac{i(a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] + 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] + \\
& i \operatorname{ArcCosh}[a x] \operatorname{Log}\left[1 + \frac{i(a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] - 2 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i a \sqrt{c}}{\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] + \\
& \frac{a \sqrt{c} \operatorname{Log}\left[\frac{2 d (i \sqrt{d} + a^2 \sqrt{c} x - i \sqrt{-a^2 c - d} \sqrt{-1 + a x} \sqrt{1 + a x})}{a \sqrt{-a^2 c - d} (\sqrt{c} + i \sqrt{d} x)}\right]}{\sqrt{-a^2 c - d}} + \frac{a \sqrt{c} \operatorname{Log}\left[\frac{2 d (-\sqrt{d} - i a^2 \sqrt{c} x + \sqrt{-a^2 c - d} \sqrt{-1 + a x} \sqrt{1 + a x})}{a \sqrt{-a^2 c - d} (i \sqrt{c} + \sqrt{d} x)}\right]}{\sqrt{-a^2 c - d}} + \\
& i \operatorname{PolyLog}\left[2, -\frac{i(-a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] - i \operatorname{PolyLog}\left[2, \frac{i(-a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] - \\
& \left. i \operatorname{PolyLog}\left[2, -\frac{i(a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] + i \operatorname{PolyLog}\left[2, \frac{i(a \sqrt{c} + \sqrt{a^2 c + d}) e^{-\operatorname{ArcCosh}[a x]}}{\sqrt{d}}\right] \right)
\end{aligned}$$

### Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f + g x} dx$$

Optimal (type 4, 785 leaves, 23 steps):

$$\begin{aligned} & -\frac{b c x \sqrt{d - c^2 d x^2}}{g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{a (1 - c^2 x^2) \sqrt{d - c^2 d x^2}}{g (1 - c x) (1 + c x)} + \frac{b \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g} - \\ & \frac{c x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{\left(1 - \frac{c^2 f^2}{g^2}\right) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \\ & \frac{(1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \frac{a \sqrt{c^2 f^2 - g^2} \sqrt{-1 + c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{g + c^2 f x}{\sqrt{c^2 f^2 - g^2} \sqrt{-1 + c^2 x^2}}\right]}{g^2 (1 - c x) (1 + c x)} + \\ & \frac{b \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\ & \frac{b \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 4, 1121 leaves):

$$\begin{aligned} & \frac{1}{2 g^2} \left( 2 a g \sqrt{d - c^2 d x^2} - 2 a c \sqrt{d} f \operatorname{ArcTan}\left[\frac{c x \sqrt{d - c^2 d x^2}}{\sqrt{d} (-1 + c^2 x^2)}\right] + \right. \\ & 2 a \sqrt{d} \sqrt{-c^2 f^2 + g^2} \operatorname{Log}[f + g x] - 2 a \sqrt{d} \sqrt{-c^2 f^2 + g^2} \operatorname{Log}[d (g + c^2 f x) + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{d - c^2 d x^2}] + \\ & \left. b \sqrt{d - c^2 d x^2} \left( \frac{2 c g x \sqrt{\frac{-1 + c x}{1 + c x}}}{1 - c x} + 2 g \operatorname{ArcCosh}[c x] + \frac{c f \sqrt{\frac{-1 + c x}{1 + c x}} \operatorname{ArcCosh}[c x]^2}{1 - c x} + \frac{1}{\sqrt{-c^2 f^2 + g^2} \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x)} - 2 (-c f + g) (c f + g) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c(f + g x)}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - \right. \\
& \left. 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c(f + g x)}}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (c f - g + i \sqrt{-c^2 f^2 + g^2}) (-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (-c f + g + i \sqrt{-c^2 f^2 + g^2}) (1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] \right) \right) \right)
\end{aligned}$$

**Problem 57: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{(f + g x)^2} dx$$

Optimal (type 4, 918 leaves, 38 steps):

$$\begin{aligned}
& - \frac{a \sqrt{d - c^2 d x^2}}{g (f + g x)} + \frac{a c^3 f^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^2 (c^2 f^2 - g^2) \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b \sqrt{-\frac{1 - c x}{1 + c x}} \sqrt{1 + c x} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g \sqrt{-1 + c x} (f + g x)} + \frac{b c^3 f^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]^2}{2 g^2 (c^2 f^2 - g^2) \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{(g + c^2 f x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c (c^2 f^2 - g^2) \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)^2} - \frac{(1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)^2} - \frac{2 a c^2 f \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c f + g} \sqrt{1 + c x}}{\sqrt{c f - g} \sqrt{-1 + c x}}\right]}{\sqrt{c f - g} g^2 \sqrt{c f + g} \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c^2 f \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{c^2 f^2 - g^2} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^2 f \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{c^2 f^2 - g^2} \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[f + g x]}{g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^2 f \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{c^2 f^2 - g^2} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^2 f \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^2 \sqrt{c^2 f^2 - g^2} \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 1154 leaves):

$$\begin{aligned}
& - \frac{a \sqrt{-d (-1 + c^2 x^2)}}{g (f + g x)} + \frac{a c \sqrt{d} \operatorname{ArcTan}\left[\frac{c x \sqrt{-d (-1 + c^2 x^2)}}{\sqrt{d} (-1 + c^2 x^2)}\right]}{g^2} + \\
& \frac{a c^2 \sqrt{d} f \operatorname{Log}[f + g x]}{g^2 \sqrt{-c^2 f^2 + g^2}} - \frac{a c^2 \sqrt{d} f \operatorname{Log}\left[d g + c^2 d f x + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{-d (-1 + c^2 x^2)}\right]}{g^2 \sqrt{-c^2 f^2 + g^2}} + \\
& \frac{1}{2 g^2} b c \sqrt{-d (-1 + c x) (1 + c x)} \left( - \frac{2 g \operatorname{ArcCosh}[c x]}{c f + c g x} + \frac{\operatorname{ArcCosh}[c x]^2}{\sqrt{-\frac{1 - c x}{1 + c x}} (1 + c x)} + \frac{2 \operatorname{Log}\left[1 + \frac{g x}{f}\right]}{\sqrt{-\frac{1 - c x}{1 + c x}} (1 + c x)} + \frac{1}{\sqrt{-c^2 f^2 + g^2} \sqrt{-\frac{1 - c x}{1 + c x}} (1 + c x)} \right) \\
& 2 c f \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) + \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[ \frac{e^{-\frac{1}{2} \text{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}} \right] + \left( \text{ArcCos} \left[ -\frac{c f}{g} \right] - \right. \\
& \left. 2 \left( \text{ArcTan} \left[ \frac{(c f + g) \text{Coth} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \text{ArcTan} \left[ \frac{(-c f + g) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \right) \text{Log} \left[ \frac{e^{\frac{1}{2} \text{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}} \right] - \\
& \left( \text{ArcCos} \left[ -\frac{c f}{g} \right] + 2 \text{ArcTan} \left[ \frac{(-c f + g) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \text{Log} \left[ \frac{(c f + g) \left( c f - g + i \sqrt{-c^2 f^2 + g^2} \right) \left( -1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)} \right] - \\
& \left( \text{ArcCos} \left[ -\frac{c f}{g} \right] - 2 \text{ArcTan} \left[ \frac{(-c f + g) \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \text{Log} \left[ \frac{(c f + g) \left( -c f + g + i \sqrt{-c^2 f^2 + g^2} \right) \left( 1 + \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)} \right] + \\
& i \left( \text{PolyLog} \left[ 2, \frac{\left( c f - i \sqrt{-c^2 f^2 + g^2} \right) \left( c f + g - i \sqrt{-c^2 f^2 + g^2} \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)} \right] - \right. \\
& \left. \text{PolyLog} \left[ 2, \frac{\left( c f + i \sqrt{-c^2 f^2 + g^2} \right) \left( c f + g - i \sqrt{-c^2 f^2 + g^2} \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh} \left[ \frac{1}{2} \text{ArcCosh}[c x] \right] \right)} \right] \right) \right)
\end{aligned}$$

**Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \text{ArcCosh}[c x])}{f + g x} dx$$

Optimal (type 4, 1270 leaves, ? steps):



$$\begin{aligned}
& - \frac{a d (c f - g) (c f + g) \sqrt{d - c^2 d x^2}}{g^3} + \frac{b c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2}}{g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^2 d (c f - g) x^2 \sqrt{d - c^2 d x^2}}{4 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{a d (2 + 3 c x - 2 c^2 x^2) \sqrt{d - c^2 d x^2}}{6 g} + \frac{b c d x (-12 - 9 c x + 4 c^2 x^2) \sqrt{d - c^2 d x^2}}{36 g \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d (c f - g) (c f + g) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^3} - \\
& \frac{a d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{2 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d (2 + 3 c x - 2 c^2 x^2) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{6 g} - \frac{b d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]^2}{4 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{c d (c f - g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 g^2} - \frac{d (c f - g) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{d (c f - g)^2 (c f + g)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^4 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \\
& \frac{d (c f - g) (c f + g) (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^2 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \frac{2 a d (c f - g)^{3/2} (c f + g)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c f + g} \sqrt{1 + c x}}{\sqrt{c f - g} \sqrt{-1 + c x}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 3068 leaves):

$$\begin{aligned}
& \sqrt{-d (-1 + c^2 x^2)} \left( \frac{a d (-3 c^2 f^2 + 4 g^2)}{3 g^3} + \frac{a c^2 d f x}{2 g^2} - \frac{a c^2 d x^2}{3 g} \right) + \frac{a c d^{3/2} f (2 c^2 f^2 - 3 g^2) \operatorname{ArcTan}\left[\frac{c x \sqrt{-d (-1 + c^2 x^2)}}{\sqrt{d} (-1 + c^2 x^2)}\right]}{2 g^4} + \\
& \frac{a d^{3/2} (-c^2 f^2 + g^2)^{3/2} \operatorname{Log}[f + g x]}{g^4} - \frac{a d^{3/2} (-c^2 f^2 + g^2)^{3/2} \operatorname{Log}[d g + c^2 d f x + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{-d (-1 + c^2 x^2)}]}{g^4} +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2g^2} b d \sqrt{-d(-1+cx)(1+cx)} \left( -\frac{2cgx}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + 2g \operatorname{ArcCosh}[cx] - \frac{cf \operatorname{ArcCosh}[cx]^2}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + \frac{1}{\sqrt{-c^2f^2+g^2} \sqrt{\frac{-1+cx}{1+cx}}(1+cx)} - 2(-cf+g)(cf+g) \right. \\
& \left. \left( 2 \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\frac{(cf+g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] - 2i \operatorname{ArcCos}\left[-\frac{cf}{g}\right] \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] + \right. \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(cf+g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[cx]} \sqrt{-c^2f^2+g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf+cgx}}\right] + \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] - \right. \\
& \left. 2 \left( \operatorname{ArcTan}\left[\frac{(cf+g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[cx]} \sqrt{-c^2f^2+g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf+cgx}}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) \operatorname{Log}\left[\frac{(cf+g)(cf-g+i\sqrt{-c^2f^2+g^2})(-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g(cf+g+i\sqrt{-c^2f^2+g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) \operatorname{Log}\left[\frac{(cf+g)(-cf+g+i\sqrt{-c^2f^2+g^2})(1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g(cf+g+i\sqrt{-c^2f^2+g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(cf-i\sqrt{-c^2f^2+g^2})(cf+g-i\sqrt{-c^2f^2+g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g(cf+g+i\sqrt{-c^2f^2+g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] - \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(cf+i\sqrt{-c^2f^2+g^2})(cf+g-i\sqrt{-c^2f^2+g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g(cf+g+i\sqrt{-c^2f^2+g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72 \sqrt{\frac{-1+cx}{1+cx}} (1+cx)} b d \sqrt{-d(-1+cx)(1+cx)} \left( -\frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 \left( -2 \operatorname{ArcCosh}[cx] \operatorname{ArcTan}\left[\frac{(cf+g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \\
& 2i \operatorname{ArcCos}\left[-\frac{cf}{g}\right] \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \left. \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] + \right. \right. \\
& \left. \left. 2 \left( \operatorname{ArcTan}\left[\frac{(cf+g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf+cgx}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{(cf+g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf+cgx}}\right] + \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
& \operatorname{Log}\left[\frac{(cf+g) (cf-g+i\sqrt{-c^2 f^2 + g^2}) (-1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g (cf+g+i\sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] + \\
& \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{(-cf+g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(cf+g) (-cf+g+i\sqrt{-c^2 f^2 + g^2}) (1+\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g (cf+g+i\sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] - \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(cf-i\sqrt{-c^2 f^2 + g^2}) (cf+g-i\sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g (cf+g+i\sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(cf+i\sqrt{-c^2 f^2 + g^2}) (cf+g-i\sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}{g (cf+g+i\sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[cx]\right])}\right] \right) - \\
& \frac{1}{g^4} \left( -18cg(-4c^2 f^2 + g^2)x + 18g(-4c^2 f^2 + g^2) \sqrt{\frac{-1+cx}{1+cx}} (1+cx) \operatorname{ArcCosh}[cx] + 18cf(2c^2 f^2 - g^2) \operatorname{ArcCosh}[cx]^2 - \right.
\end{aligned}$$

$$\begin{aligned}
& 9 c f g^2 \operatorname{Cosh}[2 \operatorname{ArcCosh}[c x]] + 2 g^3 \operatorname{Cosh}[3 \operatorname{ArcCosh}[c x]] + \frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 (8 c^4 f^4 - 8 c^2 f^2 g^2 + g^4) \\
& \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \\
& \left. \left. \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \right. \\
& \left. \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (c f - g + i \sqrt{-c^2 f^2 + g^2}) (-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \left( \operatorname{ArcCos}\left[ \right. \right. \\
& \left. \left. -\frac{c f}{g} \right] - 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (-c f + g + i \sqrt{-c^2 f^2 + g^2}) (1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \operatorname{PolyLog}\left[2, \right. \right. \\
& \left. \left. \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] \right) + \\
& \left. \left. 18 c f g^2 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[2 \operatorname{ArcCosh}[c x]] - 6 g^3 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[3 \operatorname{ArcCosh}[c x]] \right) \right)
\end{aligned}$$

**Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{f + g x} dx$$

Optimal (type 4, 1744 leaves, 39 steps):

$$\begin{aligned} & \frac{2 b c d^2 x \sqrt{d - c^2 d x^2}}{15 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c d^2 (c^2 f^2 - 2 g^2) x \sqrt{d - c^2 d x^2}}{3 g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c d^2 (c^2 f^2 - g^2)^2 x \sqrt{d - c^2 d x^2}}{g^5 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^3 d^2 f x^2 \sqrt{d - c^2 d x^2}}{16 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\ & \frac{b c^3 d^2 f (c^2 f^2 - 2 g^2) x^2 \sqrt{d - c^2 d x^2}}{4 g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d^2 x^3 \sqrt{d - c^2 d x^2}}{45 g \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^3 d^2 (c^2 f^2 - 2 g^2) x^3 \sqrt{d - c^2 d x^2}}{9 g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^5 d^2 f x^4 \sqrt{d - c^2 d x^2}}{16 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\ & \frac{b c^5 d^2 x^5 \sqrt{d - c^2 d x^2}}{25 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{a d^2 (c^2 f^2 - g^2)^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2}}{g^5 (1 - c x) (1 + c x)} + \frac{b d^2 (c^2 f^2 - g^2)^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^5} + \\ & \frac{c^2 d^2 f x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{8 g^2} - \frac{c^2 d^2 f (c^2 f^2 - 2 g^2) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 g^4} - \\ & \frac{c^4 d^2 f x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{4 g^2} - \frac{2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{15 g} - \\ & \frac{d^2 (c^2 f^2 - 2 g^2) (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 g^3} - \frac{c^2 d^2 x^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{5 g} + \\ & \frac{c d^2 f \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{16 b g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c d^2 f (c^2 f^2 - 2 g^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\ & \frac{c d^2 (c^2 f^2 - g^2)^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g^5 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{d^2 (c^2 f^2 - g^2)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^6 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \\ & \frac{d^2 (c^2 f^2 - g^2)^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^4 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \frac{a d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{-1 + c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{g + c^2 f x}{\sqrt{c^2 f^2 - g^2} \sqrt{-1 + c^2 x^2}}\right]}{g^6 (1 - c x) (1 + c x)} + \\ & \frac{b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\ & \frac{b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 4, 7300 leaves):

$$\begin{aligned}
& \sqrt{-d(-1+c^2x^2)} \left( \frac{ad^2(15c^4f^4-35c^2f^2g^2+23g^4)}{15g^5} - \frac{ac^2d^2f(4c^2f^2-9g^2)x}{8g^4} - \frac{ac^2d^2(-5c^2f^2+11g^2)x^2}{15g^3} - \frac{ac^4d^2fx^3}{4g^2} + \frac{ac^4d^2x^4}{5g} \right) - \\
& \frac{acd^{5/2}f(8c^4f^4-20c^2f^2g^2+15g^4)\operatorname{ArcTan}\left[\frac{cx\sqrt{-d(-1+c^2x^2)}}{\sqrt{d}(-1+c^2x^2)}\right]}{8g^6} + \frac{ad^{5/2}(-c^2f^2+g^2)^{5/2}\operatorname{Log}[f+gx]}{g^6} - \\
& \frac{ad^{5/2}(-c^2f^2+g^2)^{5/2}\operatorname{Log}\left[dg+c^2dfx+\sqrt{d}\sqrt{-c^2f^2+g^2}\sqrt{-d(-1+c^2x^2)}\right]}{g^6} + \\
& \frac{1}{2g^2}bd^2\sqrt{-d(-1+cx)(1+cx)} \left( -\frac{2cgx}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + 2g\operatorname{ArcCosh}[cx] - \frac{cf\operatorname{ArcCosh}[cx]^2}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{-c^2f^2+g^2}} + \frac{1}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} - 2(-cf+g) \right. \\
& \left. (cf+g) \left( 2\operatorname{ArcCosh}[cx]\operatorname{ArcTan}\left[\frac{(cf+g)\operatorname{Coth}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] - 2i\operatorname{ArcCos}\left[-\frac{cf}{g}\right]\operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2\left(\operatorname{ArcTan}\left[\frac{(cf+g)\operatorname{Coth}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right]\right) \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2}\operatorname{ArcCosh}[cx]}\sqrt{-c^2f^2+g^2}}{\sqrt{2}\sqrt{g}\sqrt{cf+cgx}}\right] + \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] - \right. \\
& \left. 2\left(\operatorname{ArcTan}\left[\frac{(cf+g)\operatorname{Coth}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right]\right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2}\operatorname{ArcCosh}[cx]}\sqrt{-c^2f^2+g^2}}{\sqrt{2}\sqrt{g}\sqrt{cf+cgx}}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2\operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) \operatorname{Log}\left[\frac{(cf+g)(cf-g+i\sqrt{-c^2f^2+g^2})(-1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right])}{g(cf+g+i\sqrt{-c^2f^2+g^2})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{cf}{g}\right] - 2\operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] \right) \operatorname{Log}\left[\frac{(cf+g)(-cf+g+i\sqrt{-c^2f^2+g^2})(1+\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right])}{g(cf+g+i\sqrt{-c^2f^2+g^2})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}\right] +
\end{aligned}$$

$$\begin{aligned}
& i \left( \text{PolyLog}\left[2, \frac{\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{\left(c f + i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] \right) - \\
& \frac{1}{36 \sqrt{\frac{-1+c x}{1+c x}} (1+c x)} b d^2 \sqrt{-d(-1+c x)(1+c x)} \left( -\frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 \left( -2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \\
& 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \\
& \left. \left. \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - \right. \\
& \left. 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) \left(c f - g + i \sqrt{-c^2 f^2 + g^2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] + \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) \left(-c f + g + i \sqrt{-c^2 f^2 + g^2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \\
& i \left( \text{PolyLog}\left[2, \frac{\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \text{PolyLog}\left[2, \frac{\left(c f + i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] \right) \right) - \\
& \frac{1}{g^4} \left( -18 c g \left(-4 c^2 f^2 + g^2\right) x + 18 g \left(-4 c^2 f^2 + g^2\right) \sqrt{\frac{-1 + c x}{1 + c x}} \left(1 + c x\right) \operatorname{ArcCosh}[c x] + 18 c f \left(2 c^2 f^2 - g^2\right) \operatorname{ArcCosh}[c x]^2 - \right. \\
& 9 c f g^2 \operatorname{Cosh}\left[2 \operatorname{ArcCosh}[c x]\right] + 2 g^3 \operatorname{Cosh}\left[3 \operatorname{ArcCosh}[c x]\right] + \frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 \left(8 c^4 f^4 - 8 c^2 f^2 g^2 + g^4\right) \\
& \left. \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{\left(c f + g\right) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{\left(-c f + g\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{\left(c f + g\right) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{\left(-c f + g\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \\
& \left. \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{\left(c f + g\right) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTan}\left[\frac{\left(-c f + g\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \right. \right. \\
& \left. \left. \operatorname{ArcTan}\left[\frac{\left(-c f + g\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{\left(c f + g\right) \left(c f - g + i \sqrt{-c^2 f^2 + g^2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \left( \operatorname{ArcCos}\left[ \right. \right. \\
& \left. \left. -\frac{c f}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{\left(-c f + g\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{\left(c f + g\right) \left(-c f + g + i \sqrt{-c^2 f^2 + g^2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] + \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \operatorname{PolyLog}\left[2, \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \left. \left. \frac{\left( c f + i \sqrt{-c^2 f^2 + g^2} \right) \left( c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)} \right] \right) + \right. \\
& \left. \left. \left. 18 c f g^2 \operatorname{ArcCosh} [c x] \operatorname{Sinh} [2 \operatorname{ArcCosh} [c x]] - 6 g^3 \operatorname{ArcCosh} [c x] \operatorname{Sinh} [3 \operatorname{ArcCosh} [c x]] \right] \right) - \right. \\
& b d^2 \left( \frac{1}{32 g^2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \sqrt{-d (-1+c x) (1+c x)} \left( -2 c g x + 2 g \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh} [c x] - c f \operatorname{ArcCosh} [c x]^2 + \right. \right. \\
& \left. \frac{1}{\sqrt{-c^2 f^2 + g^2}} (-2 c^2 f^2 + g^2) \left( 2 \operatorname{ArcCosh} [c x] \operatorname{ArcTan} \left[ \frac{(c f + g) \operatorname{Coth} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] - \right. \right. \\
& \left. \left. 2 i \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \left( \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] + 2 \right. \right. \right. \\
& \left. \left. \left. \left( \operatorname{ArcTan} \left[ \frac{(c f + g) \operatorname{Coth} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \right) \operatorname{Log} \left[ \frac{e^{-\frac{1}{2} \operatorname{ArcCosh} [c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}} \right] + \right. \\
& \left. \left. \left. \left( \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] - 2 \left( \operatorname{ArcTan} \left[ \frac{(c f + g) \operatorname{Coth} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \right) \right) \right. \\
& \left. \left. \operatorname{Log} \left[ \frac{e^{\frac{1}{2} \operatorname{ArcCosh} [c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}} \right] - \left( \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] + 2 \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \right) \right. \\
& \left. \left. \operatorname{Log} \left[ \frac{(c f + g) \left( c f - g + i \sqrt{-c^2 f^2 + g^2} \right) \left( -1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)} \right] - \left( \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] - 2 \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \operatorname{Log}\left[\frac{(c f + g) \left(-c f + g + i \sqrt{-c^2 f^2 + g^2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \operatorname{PolyLog}\left[2, \right. \right. \\
& \left. \left. \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] \right) + \\
& \frac{1}{16 \sqrt{-c^2 f^2 + g^2} \sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \sqrt{-d(-1+c x)(1+c x)} \left( -2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \\
& \left. 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
& \operatorname{Log}\left[\frac{(c f + g) \left(c f - g + i \sqrt{-c^2 f^2 + g^2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] +
\end{aligned}$$

$$\begin{aligned}
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) \left(-c f + g + i \sqrt{-c^2 f^2 + g^2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] \right) + \\
& \frac{1}{144 g^4 \sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \sqrt{-d (-1+c x) (1+c x)} \left( -18 c g (-4 c^2 f^2 + g^2) x + 18 g (-4 c^2 f^2 + g^2) \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x] + \right. \\
& \left. 18 c f (2 c^2 f^2 - g^2) \operatorname{ArcCosh}[c x]^2 - 9 c f g^2 \operatorname{Cosh}[2 \operatorname{ArcCosh}[c x]] + 2 g^3 \operatorname{Cosh}[3 \operatorname{ArcCosh}[c x]] + \right. \\
& \left. \frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 (8 c^4 f^4 - 8 c^2 f^2 g^2 + g^4) \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
& \left. \left. 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \right. \right. \right. \\
& \left. \left. \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Log}\left[\frac{(c f + g) \left(c f - g + i \sqrt{-c^2 f^2 + g^2}\right) \left(-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2\right. \\
& \left. \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right]\right) \operatorname{Log}\left[\frac{(c f + g) \left(-c f + g + i \sqrt{-c^2 f^2 + g^2}\right) \left(1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right] - \operatorname{PolyLog}\left[2, \right. \right. \\
& \left. \left. \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}{g \left(c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]\right)}\right]\right) + \\
& \left. 18 c f g^2 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[2 \operatorname{ArcCosh}[c x]] - 6 g^3 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[3 \operatorname{ArcCosh}[c x]]\right] - \frac{1}{32 \sqrt{\frac{-1+c x}{1+c x}} (1+c x)} \\
& \sqrt{-d (-1+c x) (1+c x)} \left( -\frac{2 c \left(16 c^4 f^4 - 12 c^2 f^2 g^2 + g^4\right) x}{g^5} + \frac{32 c^4 f^4 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{g^5} - \right. \\
& \frac{24 c^2 f^2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{g^3} + \frac{2 \sqrt{\frac{-1+c x}{1+c x}} (1+c x) \operatorname{ArcCosh}[c x]}{g} - \frac{16 c^5 f^5 \operatorname{ArcCosh}[c x]^2}{g^6} + \\
& \frac{16 c^3 f^3 \operatorname{ArcCosh}[c x]^2}{g^4} - \frac{3 c f \operatorname{ArcCosh}[c x]^2}{g^2} - \frac{2 c f \left(-2 c^2 f^2 + g^2\right) \operatorname{Cosh}[2 \operatorname{ArcCosh}[c x]]}{g^4} - \\
& \left. \frac{8 c^2 f^2 \operatorname{Cosh}[3 \operatorname{ArcCosh}[c x]]}{9 g^3} + \frac{2 \operatorname{Cosh}[3 \operatorname{ArcCosh}[c x]]}{9 g} + \frac{c f \operatorname{Cosh}[4 \operatorname{ArcCosh}[c x]]}{4 g^2} - \frac{2 \operatorname{Cosh}[5 \operatorname{ArcCosh}[c x]]}{25 g} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{g^6 \sqrt{-c^2 f^2 + g^2}} (-2 c^2 f^2 + g^2) (16 c^4 f^4 - 16 c^2 f^2 g^2 + g^4) \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \\
& 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \right. \right. \\
& \left. \left. \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{(c f + g) (c f - g + i \sqrt{-c^2 f^2 + g^2}) (-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \right. \\
& \left. \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (-c f + g + i \sqrt{-c^2 f^2 + g^2}) (1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \operatorname{PolyLog}\left[2, \right. \right. \\
& \left. \left. \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] \right) \right) - \frac{8 c^3 f^3 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[2 \operatorname{ArcCosh}[c x]]}{g^4} + \\
& \frac{4 c f \operatorname{ArcCosh}[c x] \operatorname{Sinh}[2 \operatorname{ArcCosh}[c x]]}{g^2} + \frac{8 c^2 f^2 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[3 \operatorname{ArcCosh}[c x]]}{3 g^3} - \frac{2 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[3 \operatorname{ArcCosh}[c x]]}{3 g} -
\end{aligned}$$

$$\left. \left. \frac{c f \operatorname{ArcCosh}[c x] \operatorname{Sinh}[4 \operatorname{ArcCosh}[c x]]}{g^2} + \frac{2 \operatorname{ArcCosh}[c x] \operatorname{Sinh}[5 \operatorname{ArcCosh}[c x]]}{5 g} \right) \right)$$

**Problem 69:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(f + g x) \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 4, 365 leaves, 10 steps):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}} - \frac{\sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}} +$$

$$\frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}}$$

Result (type 4, 932 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-c^2 f^2 + g^2}} \left( \frac{a \operatorname{Log}[f + g x]}{\sqrt{d}} - \frac{a \operatorname{Log}[d (g + c^2 f x) + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{d - c^2 d x^2}]}{\sqrt{d}} - \frac{1}{\sqrt{d - c^2 d x^2}} \right. \\
& \text{b } \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) + \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - \right. \\
& \left. 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (c f - g + i \sqrt{-c^2 f^2 + g^2}) (-1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \\
& \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{(c f + g) (-c f + g + i \sqrt{-c^2 f^2 + g^2}) (1 + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] + \\
& \text{i } \left( \operatorname{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right])}\right] \right) \right)
\end{aligned}$$

**Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(f + g x)^2 \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 4, 523 leaves, 13 steps):

$$\begin{aligned} & - \frac{g \sqrt{-1 + c x} \sqrt{-\frac{1 - c x}{1 + c x}} (1 + c x)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{(c^2 f^2 - g^2) (f + g x) \sqrt{d - c^2 d x^2}} + \frac{c^2 f \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} \\ & - \frac{c^2 f \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} + \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[f + g x]}{(c^2 f^2 - g^2) \sqrt{d - c^2 d x^2}} + \\ & - \frac{b c^2 f \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} - \frac{b c^2 f \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} \end{aligned}$$

Result (type 4, 1115 leaves):

$$\begin{aligned} & - \frac{a g \sqrt{d - c^2 d x^2}}{d (-c^2 f^2 + g^2) (f + g x)} - \frac{a c^2 f \operatorname{Log}[f + g x]}{\sqrt{d} (-c^2 f^2 + g^2)^{3/2}} - \frac{a c^2 f \operatorname{Log}\left[d (g + c^2 f x) + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{d - c^2 d x^2}\right]}{\sqrt{d} (c f - g) (c f + g) \sqrt{-c^2 f^2 + g^2}} + \\ & \frac{1}{\sqrt{d - c^2 d x^2}} b c \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \left( - \frac{g \sqrt{\frac{-1 + c x}{1 + c x}} (1 + c x) \operatorname{ArcCosh}[c x]}{(c f - g) (c f + g) (c f + c g x)} + \frac{\operatorname{Log}\left[1 + \frac{g x}{f}\right]}{c^2 f^2 - g^2} + \right. \\ & \left. \frac{1}{(-c^2 f^2 + g^2)^{3/2}} c f \left( 2 \operatorname{ArcCosh}[c x] \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 i \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right. \\ & \left. \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{(c f + g) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \operatorname{ArcTan}\left[\frac{(-c f + g) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \right) \\ & \left. \operatorname{Log}\left[\frac{e^{-\frac{1}{2} \operatorname{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}}\right] + \left( \operatorname{ArcCos}\left[-\frac{c f}{g}\right] - \right. \right. \end{aligned}$$



$$\begin{aligned}
& 2 \left( \operatorname{ArcTan} \left[ \frac{(c f + g) \operatorname{Coth} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \operatorname{Log} \left[ \frac{e^{\frac{1}{2} \operatorname{ArcCosh} [c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}} \right] - \\
& \left( \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] + 2 \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \operatorname{Log} \left[ \frac{(c f + g) \left( c f - g + i \sqrt{-c^2 f^2 + g^2} \right) \left( -1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)} \right] - \\
& \left( \operatorname{ArcCos} \left[ -\frac{c f}{g} \right] - 2 \operatorname{ArcTan} \left[ \frac{(-c f + g) \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \operatorname{Log} \left[ \frac{(c f + g) \left( -c f + g + i \sqrt{-c^2 f^2 + g^2} \right) \left( 1 + \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)} \right] + \\
& i \left( \operatorname{PolyLog} \left[ 2, \frac{\left( c f - i \sqrt{-c^2 f^2 + g^2} \right) \left( c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)} \right] - \right. \\
& \left. \operatorname{PolyLog} \left[ 2, \frac{\left( c f + i \sqrt{-c^2 f^2 + g^2} \right) \left( c f + g - i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)}{g \left( c f + g + i \sqrt{-c^2 f^2 + g^2} \operatorname{Tanh} \left[ \frac{1}{2} \operatorname{ArcCosh} [c x] \right] \right)} \right] \right) \right)
\end{aligned}$$

**Problem 74: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh} [c x]}{(f + g x) (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 773 leaves, 27 steps):

$$\begin{aligned}
& - \frac{(1-cx)(a+b \operatorname{ArcCosh}[cx])}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)(a+b \operatorname{ArcCosh}[cx])}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1+cx}(a+b \operatorname{ArcCosh}[cx])\operatorname{Log}\left[1+\frac{e^{\operatorname{ArcCosh}[cx]}g}{cf-\sqrt{c^2f^2-g^2}}\right]}{d(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}} + \\
& \frac{g^2\sqrt{-1+cx}\sqrt{1+cx}(a+b \operatorname{ArcCosh}[cx])\operatorname{Log}\left[1+\frac{e^{\operatorname{ArcCosh}[cx]}g}{cf+\sqrt{c^2f^2-g^2}}\right]}{d(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}} + \frac{b\sqrt{(1-cx)(1+cx)}\sqrt{1-c^2x^2}\operatorname{Log}\left[\sqrt{-\frac{1-cx}{1+cx}}\right]}{d(cf+g)\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}} - \\
& \frac{b\sqrt{(1-cx)(1+cx)}\sqrt{1-c^2x^2}\operatorname{Log}\left[\frac{2}{1+cx}\right]}{2d(cf-g)\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}} - \frac{b\sqrt{(1-cx)(1+cx)}\sqrt{1-c^2x^2}\operatorname{Log}\left[\frac{2}{1+cx}\right]}{2d(cf+g)\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}} - \\
& \frac{bg^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[cx]}g}{cf-\sqrt{c^2f^2-g^2}}\right]}{d(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}} + \frac{bg^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[cx]}g}{cf+\sqrt{c^2f^2-g^2}}\right]}{d(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}
\end{aligned}$$

Result (type 4, 1386 leaves):

$$\begin{aligned}
& \frac{(-ag+ac^2fx)\sqrt{-d(-1+c^2x^2)}}{d^2(-c^2f^2+g^2)(-1+c^2x^2)} + \frac{ag^2\operatorname{Log}[f+gx]}{d^{3/2}(-cf+g)(cf+g)\sqrt{-c^2f^2+g^2}} - \frac{ag^2\operatorname{Log}\left[dg+c^2dfx+\sqrt{d}\sqrt{-c^2f^2+g^2}\sqrt{-d(-1+c^2x^2)}\right]}{d^{3/2}(-cf+g)(cf+g)\sqrt{-c^2f^2+g^2}} - \\
& \frac{1}{d} \left( - \frac{\sqrt{-\frac{1+cx}{1+cx}}(1+cx)\operatorname{ArcCosh}[cx]\operatorname{Coth}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{2(cf+g)\sqrt{-d(-1+cx)}(1+cx)} + \frac{\sqrt{-\frac{1+cx}{1+cx}}(1+cx)\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]\right]}{(cf-g)\sqrt{-d(-1+cx)}(1+cx)} + \right. \\
& \left. \frac{\sqrt{-\frac{1+cx}{1+cx}}(1+cx)\operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]\right]}{(cf+g)\sqrt{-d(-1+cx)}(1+cx)} + \frac{1}{(-cf+g)(cf+g)\sqrt{-c^2f^2+g^2}\sqrt{-d(-1+cx)}(1+cx)} g^2\sqrt{-\frac{1+cx}{1+cx}}(1+cx) \right. \\
& \left. \left( 2\operatorname{ArcCosh}[cx]\operatorname{ArcTan}\left[\frac{(cf+g)\operatorname{Coth}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] - 2i\operatorname{ArcCos}\left[-\frac{cf}{g}\right]\operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] + \left(\operatorname{ArcCos}\left[-\frac{cf}{g}\right] + \right. \right. \\
& \left. \left. 2i\left(-i\operatorname{ArcTan}\left[\frac{(cf+g)\operatorname{Coth}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right] - i\operatorname{ArcTan}\left[\frac{(-cf+g)\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}[cx]\right]}{\sqrt{-c^2f^2+g^2}}\right]\right) \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2}\operatorname{ArcCosh}[cx]}\sqrt{-c^2f^2+g^2}}{\sqrt{2}\sqrt{g}\sqrt{cf+cgx}}\right] +
\end{aligned}$$

$$\begin{aligned}
& \left( \text{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \left( -i \text{ArcTan}\left[\frac{(c f + g) \text{Coth}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - i \text{ArcTan}\left[\frac{(-c f + g) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \text{Log}\left[\frac{e^{\frac{1}{2} \text{ArcCosh}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \left( \text{ArcCos}\left[-\frac{c f}{g}\right] - 2 \text{ArcTan}\left[\frac{(-c f + g) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
& \text{Log}\left[1 - \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] + \left( -\text{ArcCos}\left[-\frac{c f}{g}\right] - \right. \\
& \left. 2 \text{ArcTan}\left[\frac{(-c f + g) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \text{Log}\left[1 - \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] + \\
& i \left( \text{PolyLog}\left[2, \frac{(c f - i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] - \text{PolyLog}\left[2, \right. \right. \\
& \left. \left. \frac{(c f + i \sqrt{-c^2 f^2 + g^2}) (c f + g - i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}{g (c f + g + i \sqrt{-c^2 f^2 + g^2} \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right])}\right] \right) - \frac{\sqrt{\frac{-1+c x}{1+c x}} (1+c x) \text{ArcCosh}[c x] \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{2 (c f - g) \sqrt{-d (-1+c x) (1+c x)}}
\end{aligned}$$

**Problem 79: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \text{ArcCosh}[c x])^2 \text{Log}[h (f + g x)^m]}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 4, 774 leaves, 14 steps):

$$\begin{aligned}
& \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^4}{12 b^2 c \sqrt{1-c^2 x^2}} - \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^3 \operatorname{Log}\left[1+\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{3 b c \sqrt{1-c^2 x^2}} - \\
& \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^3 \operatorname{Log}\left[1+\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{3 b c \sqrt{1-c^2 x^2}} + \frac{\sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^3 \operatorname{Log}[h(f+gx)^m]}{3 b c \sqrt{1-c^2 x^2}} - \\
& \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^2 \operatorname{PolyLog}\left[2,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} - \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^2 \operatorname{PolyLog}\left[2,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} + \\
& \frac{2 b m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[3,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} + \\
& \frac{2 b m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[3,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} - \\
& \frac{2 b^2 m \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[4,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} - \frac{2 b^2 m \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[4,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 80: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a+b \operatorname{ArcCosh}[cx]) \operatorname{Log}[h(f+gx)^m]}{\sqrt{1-c^2 x^2}} dx$$

Optimal (type 4, 600 leaves, 12 steps):

$$\begin{aligned}
& \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^3}{6 b^2 c \sqrt{1-c^2 x^2}} - \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^2 \operatorname{Log}\left[1+\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{2 b c \sqrt{1-c^2 x^2}} \\
& \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^2 \operatorname{Log}\left[1+\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{2 b c \sqrt{1-c^2 x^2}} + \frac{\sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^2 \operatorname{Log}[h(f+gx)^m]}{2 b c \sqrt{1-c^2 x^2}} \\
& \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[2,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} - \frac{m \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx]) \operatorname{PolyLog}\left[2,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} \\
& \frac{b m \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[3,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}} + \frac{b m \sqrt{-1+cx} \sqrt{1+cx} \operatorname{PolyLog}\left[3,-\frac{e^{\operatorname{ArcCosh}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 81: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Log}[h(f+gx)^m]}{\sqrt{1-c^2 x^2}} dx$$

Optimal (type 4, 237 leaves, 9 steps):

$$\begin{aligned}
& \frac{i m \operatorname{ArcSin}[cx]^2}{2 c} - \frac{m \operatorname{ArcSin}[cx] \operatorname{Log}\left[1-\frac{i e^{i \operatorname{ArcSin}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c} - \frac{m \operatorname{ArcSin}[cx] \operatorname{Log}\left[1-\frac{i e^{i \operatorname{ArcSin}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c} \\
& \frac{\operatorname{ArcSin}[cx] \operatorname{Log}[h(f+gx)^m]}{c} + \frac{i m \operatorname{PolyLog}\left[2,\frac{i e^{i \operatorname{ArcSin}[cx]} g}{c f-\sqrt{c^2 f^2-g^2}}\right]}{c} + \frac{i m \operatorname{PolyLog}\left[2,\frac{i e^{i \operatorname{ArcSin}[cx]} g}{c f+\sqrt{c^2 f^2-g^2}}\right]}{c}
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 87: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcCosh}[a+bx]}{x} dx$$

Optimal (type 4, 131 leaves, 9 steps):

$$-\frac{1}{2} \operatorname{ArcCosh}[a + b x]^2 + \operatorname{ArcCosh}[a + b x] \operatorname{Log}\left[1 - \frac{e^{\operatorname{ArcCosh}[a + b x]}}{a - \sqrt{-1 + a^2}}\right] +$$

$$\operatorname{ArcCosh}[a + b x] \operatorname{Log}\left[1 - \frac{e^{\operatorname{ArcCosh}[a + b x]}}{a + \sqrt{-1 + a^2}}\right] + \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcCosh}[a + b x]}}{a - \sqrt{-1 + a^2}}\right] + \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcCosh}[a + b x]}}{a + \sqrt{-1 + a^2}}\right]$$

Result (type 4, 221 leaves):

$$\frac{1}{2} \operatorname{ArcCosh}[a + b x]^2 - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - a}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(1 + a) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[a + b x]\right]}{\sqrt{-1 + a^2}}\right] +$$

$$\left(\operatorname{ArcCosh}[a + b x] + 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - a}}{\sqrt{2}}\right]\right) \operatorname{Log}\left[1 + \left(-a + \sqrt{-1 + a^2}\right) e^{-\operatorname{ArcCosh}[a + b x]}\right] +$$

$$\left(\operatorname{ArcCosh}[a + b x] - 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - a}}{\sqrt{2}}\right]\right) \operatorname{Log}\left[1 - \left(a + \sqrt{-1 + a^2}\right) e^{-\operatorname{ArcCosh}[a + b x]}\right] -$$

$$\operatorname{PolyLog}\left[2, \left(a - \sqrt{-1 + a^2}\right) e^{-\operatorname{ArcCosh}[a + b x]}\right] - \operatorname{PolyLog}\left[2, \left(a + \sqrt{-1 + a^2}\right) e^{-\operatorname{ArcCosh}[a + b x]}\right]$$

**Problem 88: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcCosh}[a + b x]}{x^2} dx$$

Optimal (type 3, 64 leaves, 4 steps):

$$-\frac{\operatorname{ArcCosh}[a + b x]}{x} - \frac{2 b \operatorname{ArcTan}\left[\frac{\sqrt{1 - a} \sqrt{1 + b x}}{\sqrt{1 + a} \sqrt{-1 + b x}}\right]}{\sqrt{1 - a^2}}$$

Result (type 3, 83 leaves):

$$-\frac{\operatorname{ArcCosh}[a + b x]}{x} - \frac{i b \operatorname{Log}\left[\frac{2 \left(\sqrt{-1 + b x} \sqrt{1 + b x} + \frac{i(-1 + a^2 - b x)}{\sqrt{1 - a^2}}\right)}{b x}\right]}{\sqrt{1 - a^2}}$$

**Problem 89: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcCosh}[a + b x]}{x^3} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$\frac{b \sqrt{-1+a+bx} \sqrt{1+a+bx}}{2(1-a^2)x} - \frac{\text{ArcCosh}[a+bx]}{2x^2} - \frac{ab^2 \text{ArcTan}\left[\frac{\sqrt{1-a}\sqrt{1+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right]}{(1-a^2)^{3/2}}$$

Result (type 3, 136 leaves):

$$-\text{ArcCosh}[a+bx] + \frac{bx \left( -\sqrt{-1+a+bx} \sqrt{1+a+bx} + \frac{iabx \log\left[\frac{4i\sqrt{1-a^2}(-1+a^2+bx-i\sqrt{1-a^2}\sqrt{-1+a+bx}\sqrt{1+a+bx})}{ab^2x}\right]}{\sqrt{1-a^2}} \right)}{-1+a^2} \frac{1}{2x^2}$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCosh}[a+bx]}{x^4} dx$$

Optimal (type 3, 154 leaves, 7 steps):

$$\frac{b \sqrt{-1+a+bx} \sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2 \sqrt{-1+a+bx} \sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\text{ArcCosh}[a+bx]}{3x^3} - \frac{(1+2a^2)b^3 \text{ArcTan}\left[\frac{\sqrt{1-a}\sqrt{1+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right]}{3(1-a^2)^{5/2}}$$

Result (type 3, 162 leaves):

$$\frac{1}{6} \left( \frac{b \sqrt{-1+a+bx} \sqrt{1+a+bx} (1-a^2+3abx)}{(-1+a^2)^2x^2} - \frac{2 \text{ArcCosh}[a+bx]}{x^3} - \frac{i(1+2a^2)b^3 \log\left[\frac{12(1-a^2)^{3/2}(-i+i a^2+iabx+\sqrt{1-a^2}\sqrt{-1+a+bx}\sqrt{1+a+bx})}{b^3(x+2a^2x)}\right]}{(1-a^2)^{5/2}} \right)$$

Problem 125: Result more than twice size of optimal antiderivative.

$$\int (a+bx \text{ArcCosh}[c+dx])^4 dx$$

Optimal (type 3, 129 leaves, 6 steps):

$$24b^4x - \frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{ArcCosh}[c+dx])}{d} + \frac{12b^2(c+dx)(a+b\text{ArcCosh}[c+dx])^2}{d} - \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{ArcCosh}[c+dx])^3}{d} + \frac{(c+dx)(a+b\text{ArcCosh}[c+dx])^4}{d}$$

Result (type 3, 261 leaves):

$$\frac{1}{d} \left( (a^4 + 12 a^2 b^2 + 24 b^4) (c + d x) - 4 a b (a^2 + 6 b^2) \sqrt{-1 + c + d x} \sqrt{1 + c + d x} - \right. \\ \left. 4 b \left( -a^3 (c + d x) - 6 a b^2 (c + d x) + 3 a^2 b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} + 6 b^3 \sqrt{-1 + c + d x} \sqrt{1 + c + d x} \right) \text{ArcCosh}[c + d x] + \right. \\ \left. 6 b^2 \left( a^2 (c + d x) + 2 b^2 (c + d x) - 2 a b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} \right) \text{ArcCosh}[c + d x]^2 - \right. \\ \left. 4 b^3 \left( -a (c + d x) + b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} \right) \text{ArcCosh}[c + d x]^3 + b^4 (c + d x) \text{ArcCosh}[c + d x]^4 \right)$$

**Problem 127: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{ArcCosh}[c + d x])^4}{(c e + d e x)^2} dx$$

Optimal (type 4, 264 leaves, 13 steps):

$$-\frac{(a + b \text{ArcCosh}[c + d x])^4}{d e^2 (c + d x)} + \frac{8 b (a + b \text{ArcCosh}[c + d x])^3 \text{ArcTan}\left[e^{\text{ArcCosh}[c + d x]}\right]}{d e^2} - \frac{12 i b^2 (a + b \text{ArcCosh}[c + d x])^2 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[c + d x]}\right]}{d e^2} + \\ \frac{12 i b^2 (a + b \text{ArcCosh}[c + d x])^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[c + d x]}\right]}{d e^2} + \frac{24 i b^3 (a + b \text{ArcCosh}[c + d x]) \text{PolyLog}\left[3, -i e^{\text{ArcCosh}[c + d x]}\right]}{d e^2} - \\ \frac{24 i b^3 (a + b \text{ArcCosh}[c + d x]) \text{PolyLog}\left[3, i e^{\text{ArcCosh}[c + d x]}\right]}{d e^2} - \frac{24 i b^4 \text{PolyLog}\left[4, -i e^{\text{ArcCosh}[c + d x]}\right]}{d e^2} + \frac{24 i b^4 \text{PolyLog}\left[4, i e^{\text{ArcCosh}[c + d x]}\right]}{d e^2}$$

Result (type 4, 872 leaves):



$$\begin{aligned}
& \frac{1}{d e^2} \left( -\frac{a^4}{c+d x} + 4 a^3 b \left( -\frac{\operatorname{ArcCosh}[c+d x]}{c+d x} + 2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c+d x]\right]\right] \right) - \right. \\
& 6 i a^2 b^2 \left( \operatorname{ArcCosh}[c+d x] \left( -\frac{i \operatorname{ArcCosh}[c+d x]}{c+d x} + 2 \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c+d x]}\right] - 2 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c+d x]}\right] \right) + \right. \\
& \quad \left. 2 \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] - 2 \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c+d x]}\right] \right) + \\
& 4 a b^3 \left( -\frac{\operatorname{ArcCosh}[c+d x]^3}{c+d x} + 3 i \left( -\operatorname{ArcCosh}[c+d x]^2 \left( \operatorname{Log}\left[1-i e^{-\operatorname{ArcCosh}[c+d x]}\right] - \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c+d x]}\right] \right) - 2 \operatorname{ArcCosh}[c+d x] \right. \right. \\
& \quad \left. \left. \left( \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] - \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c+d x]}\right] \right) - 2 \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] + 2 \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcCosh}[c+d x]}\right] \right) \right) + \\
& b^4 \left( -\frac{7 i \pi^4}{16} + \frac{1}{2} \pi^3 \operatorname{ArcCosh}[c+d x] - \frac{3}{2} i \pi^2 \operatorname{ArcCosh}[c+d x]^2 - 2 \pi \operatorname{ArcCosh}[c+d x]^3 + i \operatorname{ArcCosh}[c+d x]^4 - \frac{\operatorname{ArcCosh}[c+d x]^4}{c+d x} + \right. \\
& \quad \frac{1}{2} \pi^3 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c+d x]}\right] - 3 i \pi^2 \operatorname{ArcCosh}[c+d x] \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c+d x]}\right] - 6 \pi \operatorname{ArcCosh}[c+d x]^2 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c+d x]}\right] + \\
& 4 i \operatorname{ArcCosh}[c+d x]^3 \operatorname{Log}\left[1+i e^{-\operatorname{ArcCosh}[c+d x]}\right] + 3 i \pi^2 \operatorname{ArcCosh}[c+d x] \operatorname{Log}\left[1-i e^{\operatorname{ArcCosh}[c+d x]}\right] + 6 \pi \operatorname{ArcCosh}[c+d x]^2 \operatorname{Log}\left[1-i e^{\operatorname{ArcCosh}[c+d x]}\right] - \\
& \quad \frac{1}{2} \pi^3 \operatorname{Log}\left[1+i e^{\operatorname{ArcCosh}[c+d x]}\right] - 4 i \operatorname{ArcCosh}[c+d x]^3 \operatorname{Log}\left[1+i e^{\operatorname{ArcCosh}[c+d x]}\right] + \frac{1}{2} \pi^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4} \left(\pi+2 i \operatorname{ArcCosh}[c+d x]\right)\right]\right] + \\
& 3 i \left(\pi-2 i \operatorname{ArcCosh}[c+d x]\right)^2 \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] - 12 i \operatorname{ArcCosh}[c+d x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c+d x]}\right] + \\
& 3 i \pi^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c+d x]}\right] + 12 \pi \operatorname{ArcCosh}[c+d x] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c+d x]}\right] + 12 \pi \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] - \\
& 24 i \operatorname{ArcCosh}[c+d x] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] + 24 i \operatorname{ArcCosh}[c+d x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c+d x]}\right] - \\
& \quad \left. 12 \pi \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c+d x]}\right] - 24 i \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcCosh}[c+d x]}\right] - 24 i \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[c+d x]}\right] \right) \Big)
\end{aligned}$$

**Problem 128: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{ArcCosh}[c+d x])^4}{(c e+d e x)^3} d x$$

Optimal (type 4, 195 leaves, 10 steps):

$$\begin{aligned}
& -\frac{2 b (a+b \operatorname{ArcCosh}[c+d x])^3}{d e^3} + \frac{2 b \sqrt{-1+c+d x} \sqrt{1+c+d x} (a+b \operatorname{ArcCosh}[c+d x])^3}{d e^3 (c+d x)} - \\
& \frac{(a+b \operatorname{ArcCosh}[c+d x])^4}{2 d e^3 (c+d x)^2} - \frac{6 b^2 (a+b \operatorname{ArcCosh}[c+d x])^2 \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c+d x]}\right]}{d e^3} + \\
& \frac{6 b^3 (a+b \operatorname{ArcCosh}[c+d x]) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c+d x]}\right]}{d e^3} + \frac{3 b^4 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcCosh}[c+d x]}\right]}{d e^3}
\end{aligned}$$

Result (type 4, 398 leaves):

$$\begin{aligned}
& \frac{1}{2 d e^3} \left( -\frac{a^4}{(c+d x)^2} + \frac{4 a^3 b \sqrt{-1+c+d x} \sqrt{1+c+d x}}{c+d x} - \frac{4 a^3 b \operatorname{ArcCosh}[c+d x]}{(c+d x)^2} - \right. \\
& \left. \frac{b^4 \operatorname{ArcCosh}[c+d x]^4}{(c+d x)^2} + 12 a^2 b^2 \left( \frac{\sqrt{\frac{-1+c+d x}{1+c+d x}} (1+c+d x) \operatorname{ArcCosh}[c+d x]}{c+d x} - \frac{\operatorname{ArcCosh}[c+d x]^2}{2 (c+d x)^2} - \operatorname{Log}[c+d x] \right) + \right. \\
& \left. 4 a b^3 \left( -\operatorname{ArcCosh}[c+d x] \left( 3 \operatorname{ArcCosh}[c+d x] - \frac{3 \sqrt{\frac{-1+c+d x}{1+c+d x}} (1+c+d x) \operatorname{ArcCosh}[c+d x]}{c+d x} + \frac{\operatorname{ArcCosh}[c+d x]^2}{(c+d x)^2} + 6 \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c+d x]}\right] \right) + \right. \\
& \left. \left. 3 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c+d x]}\right] \right) + \right. \\
& \left. 2 b^4 \left( 2 \operatorname{ArcCosh}[c+d x]^2 \left( -\operatorname{ArcCosh}[c+d x] + \frac{\sqrt{\frac{-1+c+d x}{1+c+d x}} (1+c+d x) \operatorname{ArcCosh}[c+d x]}{c+d x} - 3 \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c+d x]}\right] \right) + \right. \right. \\
& \left. \left. \left. 6 \operatorname{ArcCosh}[c+d x] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c+d x]}\right] + 3 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcCosh}[c+d x]}\right] \right) \right) \right)
\end{aligned}$$

**Problem 129: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{ArcCosh}[c+d x])^4}{(c+d e x)^4} dx$$

Optimal (type 4, 432 leaves, 21 steps):

$$\begin{aligned}
& \frac{2 b^2 (a + b \operatorname{ArcCosh}[c + d x])^2}{d e^4 (c + d x)} + \frac{2 b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} (a + b \operatorname{ArcCosh}[c + d x])^3}{3 d e^4 (c + d x)^2} - \frac{(a + b \operatorname{ArcCosh}[c + d x])^4}{3 d e^4 (c + d x)^3} - \\
& \frac{8 b^3 (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} + \frac{4 b (a + b \operatorname{ArcCosh}[c + d x])^3 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c + d x]}\right]}{3 d e^4} + \\
& \frac{4 i b^4 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} - \frac{2 i b^2 (a + b \operatorname{ArcCosh}[c + d x])^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} - \frac{4 i b^4 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} + \\
& \frac{2 i b^2 (a + b \operatorname{ArcCosh}[c + d x])^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} + \frac{4 i b^3 (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} - \\
& \frac{4 i b^3 (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} - \frac{4 i b^4 \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4} + \frac{4 i b^4 \operatorname{PolyLog}\left[4, i e^{\operatorname{ArcCosh}[c + d x]}\right]}{d e^4}
\end{aligned}$$

Result (type 4, 1374 leaves):

$$\begin{aligned}
& -\frac{a^4}{3 d e^4 (c + d x)^3} + \frac{4 a^3 b \sqrt{-1 + c + d x} \left( \frac{\sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x)}{6 (c + d x)^2} - \frac{\operatorname{ArcCosh}[c + d x]}{3 (c + d x)^3} + \frac{1}{3} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c + d x]\right]\right] \right)}{d e^4 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \sqrt{1 + c + d x}} + \\
& \left( 2 a^2 b^2 \sqrt{-1 + c + d x} \left( \frac{1}{c + d x} + \frac{\sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x]}{(c + d x)^2} - \frac{\operatorname{ArcCosh}[c + d x]^2}{(c + d x)^3} - i \operatorname{ArcCosh}[c + d x] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c + d x]}\right] + \right. \right. \\
& \left. \left. i \operatorname{ArcCosh}[c + d x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c + d x]}\right] - i \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c + d x]}\right] + i \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c + d x]}\right] \right) \right) / \\
& \left( d e^4 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \sqrt{1 + c + d x} \right) + \frac{1}{d e^4 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \sqrt{1 + c + d x}} 4 a b^3 \sqrt{-1 + c + d x}
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\operatorname{ArcCosh}[c+dx]}{c+dx} + \frac{\sqrt{\frac{-1+c+dx}{1+c+dx}} (1+c+dx) \operatorname{ArcCosh}[c+dx]^2}{2(c+dx)^2} - \frac{\operatorname{ArcCosh}[c+dx]^3}{3(c+dx)^3} - \frac{1}{2} i \left( -4 i \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCosh}[c+dx]\right]\right] \right) + \right. \\
& \left. \operatorname{ArcCosh}[c+dx]^2 \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c+dx]}\right] - \operatorname{ArcCosh}[c+dx]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c+dx]}\right] + 2 \operatorname{ArcCosh}[c+dx] \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c+dx]}\right] - \right. \\
& \left. 2 \operatorname{ArcCosh}[c+dx] \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c+dx]}\right] + 2 \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c+dx]}\right] - 2 \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcCosh}[c+dx]}\right] \right) + \\
& \frac{1}{d e^4 \sqrt{\frac{-1+c+dx}{1+c+dx}} \sqrt{1+c+dx}} b^4 \sqrt{-1+c+dx} \left( \frac{1}{2} i \left( 8 + \pi^2 - 4 i \pi \operatorname{ArcCosh}[c+dx] - 4 \operatorname{ArcCosh}[c+dx]^2 \right) \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcCosh}[c+dx]}\right] - \right. \\
& \frac{1}{96} i \left( 7 \pi^4 + 8 i \pi^3 \operatorname{ArcCosh}[c+dx] + 24 \pi^2 \operatorname{ArcCosh}[c+dx]^2 + \frac{192 i \operatorname{ArcCosh}[c+dx]^2}{c+dx} - 32 i \pi \operatorname{ArcCosh}[c+dx]^3 + \right. \\
& \frac{64 i \sqrt{\frac{-1+c+dx}{1+c+dx}} (1+c+dx) \operatorname{ArcCosh}[c+dx]^3}{(c+dx)^2} - 16 \operatorname{ArcCosh}[c+dx]^4 - \frac{32 i \operatorname{ArcCosh}[c+dx]^4}{(c+dx)^3} - \\
& 384 \operatorname{ArcCosh}[c+dx] \operatorname{Log}\left[1 - i e^{-\operatorname{ArcCosh}[c+dx]}\right] + 8 i \pi^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c+dx]}\right] + 384 \operatorname{ArcCosh}[c+dx] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c+dx]}\right] + \\
& 48 \pi^2 \operatorname{ArcCosh}[c+dx] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c+dx]}\right] - 96 i \pi \operatorname{ArcCosh}[c+dx]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c+dx]}\right] - \\
& 64 \operatorname{ArcCosh}[c+dx]^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcCosh}[c+dx]}\right] - 48 \pi^2 \operatorname{ArcCosh}[c+dx] \operatorname{Log}\left[1 - i e^{\operatorname{ArcCosh}[c+dx]}\right] + \\
& 96 i \pi \operatorname{ArcCosh}[c+dx]^2 \operatorname{Log}\left[1 - i e^{\operatorname{ArcCosh}[c+dx]}\right] - 8 i \pi^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcCosh}[c+dx]}\right] + 64 \operatorname{ArcCosh}[c+dx]^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcCosh}[c+dx]}\right] + \\
& 8 i \pi^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcCosh}[c+dx])\right]\right] + 384 \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcCosh}[c+dx]}\right] + 192 \operatorname{ArcCosh}[c+dx]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c+dx]}\right] - \\
& 48 \pi^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c+dx]}\right] + 192 i \pi \operatorname{ArcCosh}[c+dx] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c+dx]}\right] + 192 i \pi \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c+dx]}\right] + \\
& 384 \operatorname{ArcCosh}[c+dx] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcCosh}[c+dx]}\right] - 384 \operatorname{ArcCosh}[c+dx] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c+dx]}\right] - \\
& \left. 192 i \pi \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c+dx]}\right] + 384 \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcCosh}[c+dx]}\right] + 384 \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[c+dx]}\right] \right)
\end{aligned}$$

### Problem 166: Result more than twice size of optimal antiderivative.

$$\int (c e + d e x)^2 (a + b \operatorname{ArcCosh}[c + d x])^{5/2} dx$$

Optimal (type 4, 408 leaves, 26 steps):

$$\begin{aligned} & \frac{5 b^2 e^2 (c + d x) \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{6 d} + \frac{5 b^2 e^2 (c + d x)^3 \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{36 d} - \\ & \frac{5 b e^2 \sqrt{-1 + c + d x} \sqrt{1 + c + d x} (a + b \operatorname{ArcCosh}[c + d x])^{3/2}}{9 d} - \frac{5 b e^2 \sqrt{-1 + c + d x} (c + d x)^2 \sqrt{1 + c + d x} (a + b \operatorname{ArcCosh}[c + d x])^{3/2}}{18 d} + \\ & \frac{e^2 (c + d x)^3 (a + b \operatorname{ArcCosh}[c + d x])^{5/2}}{3 d} - \frac{15 b^{5/2} e^2 e^{a/b} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right]}{64 d} - \frac{5 b^{5/2} e^2 e^{\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right]}{576 d} - \\ & \frac{15 b^{5/2} e^2 e^{-\frac{a}{b}} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right]}{64 d} - \frac{5 b^{5/2} e^2 e^{-\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right]}{576 d} \end{aligned}$$

Result (type 4, 909 leaves):

$$\begin{aligned}
& \frac{1}{1728 d} \\
& e^2 \left( 432 a^2 c \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + 1620 b^2 c \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + 432 a^2 d x \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + 1620 b^2 d x \sqrt{a + b \operatorname{ArcCosh}[c + d x]} - \right. \\
& 1080 a b \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \sqrt{a + b \operatorname{ArcCosh}[c + d x]} - 1080 a b c \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \sqrt{a + b \operatorname{ArcCosh}[c + d x]} - \\
& 1080 a b d x \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + 864 a b c \operatorname{ArcCosh}[c + d x] \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + \\
& 864 a b d x \operatorname{ArcCosh}[c + d x] \sqrt{a + b \operatorname{ArcCosh}[c + d x]} - 1080 b^2 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \operatorname{ArcCosh}[c + d x] \sqrt{a + b \operatorname{ArcCosh}[c + d x]} - \\
& 1080 b^2 c \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \operatorname{ArcCosh}[c + d x] \sqrt{a + b \operatorname{ArcCosh}[c + d x]} - 1080 b^2 d x \sqrt{\frac{-1 + c + d x}{1 + c + d x}} \operatorname{ArcCosh}[c + d x] \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + \\
& 432 b^2 c \operatorname{ArcCosh}[c + d x]^2 \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + 432 b^2 d x \operatorname{ArcCosh}[c + d x]^2 \sqrt{a + b \operatorname{ArcCosh}[c + d x]} + \\
& 144 a^2 \sqrt{a + b \operatorname{ArcCosh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c + d x]] + 60 b^2 \sqrt{a + b \operatorname{ArcCosh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c + d x]] + 288 a b \operatorname{ArcCosh}[c + d x] \\
& \sqrt{a + b \operatorname{ArcCosh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c + d x]] + 144 b^2 \operatorname{ArcCosh}[c + d x]^2 \sqrt{a + b \operatorname{ArcCosh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c + d x]] - \\
& 405 b^{5/2} \sqrt{\pi} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{Erfi}\left[\frac{\sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right] - 5 b^{5/2} \sqrt{3 \pi} \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right] + \\
& 405 b^{5/2} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right] \operatorname{Sinh}\left[\frac{a}{b}\right] - 405 b^{5/2} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right] \left(\operatorname{Cosh}\left[\frac{a}{b}\right] + \operatorname{Sinh}\left[\frac{a}{b}\right]\right) + \\
& 5 b^{5/2} \sqrt{3 \pi} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right] \operatorname{Sinh}\left[\frac{3 a}{b}\right] - 5 b^{5/2} \sqrt{3 \pi} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcCosh}[c + d x]}}{\sqrt{b}}\right] \left(\operatorname{Cosh}\left[\frac{3 a}{b}\right] + \operatorname{Sinh}\left[\frac{3 a}{b}\right]\right) - \\
& \left. 120 a b \sqrt{a + b \operatorname{ArcCosh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcCosh}[c + d x]] - 120 b^2 \operatorname{ArcCosh}[c + d x] \sqrt{a + b \operatorname{ArcCosh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcCosh}[c + d x]] \right)
\end{aligned}$$

**Problem 170: Result more than twice size of optimal antiderivative.**

$$\int (c e + d e x)^2 (a + b \operatorname{ArcCosh}[c + d x])^{7/2} dx$$

Optimal (type 4, 509 leaves, 35 steps):

$$\begin{aligned}
& - \frac{175 b^3 e^2 \sqrt{-1+c+d x} \sqrt{1+c+d x} \sqrt{a+b \operatorname{ArcCosh}[c+d x]}}{54 d} - \frac{35 b^3 e^2 \sqrt{-1+c+d x} (c+d x)^2 \sqrt{1+c+d x} \sqrt{a+b \operatorname{ArcCosh}[c+d x]}}{216 d} + \\
& \frac{35 b^2 e^2 (c+d x) (a+b \operatorname{ArcCosh}[c+d x])^{3/2}}{18 d} + \frac{35 b^2 e^2 (c+d x)^3 (a+b \operatorname{ArcCosh}[c+d x])^{3/2}}{108 d} - \\
& \frac{7 b e^2 \sqrt{-1+c+d x} \sqrt{1+c+d x} (a+b \operatorname{ArcCosh}[c+d x])^{5/2}}{9 d} - \frac{7 b e^2 \sqrt{-1+c+d x} (c+d x)^2 \sqrt{1+c+d x} (a+b \operatorname{ArcCosh}[c+d x])^{5/2}}{18 d} + \\
& \frac{e^2 (c+d x)^3 (a+b \operatorname{ArcCosh}[c+d x])^{7/2}}{3 d} - \frac{105 b^{7/2} e^2 e^{a/b} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{a+b \operatorname{ArcCosh}[c+d x]}}{\sqrt{b}}\right]}{128 d} - \frac{35 b^{7/2} e^2 e^{\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{a+b \operatorname{ArcCosh}[c+d x]}}{\sqrt{b}}\right]}{3456 d} + \\
& \frac{105 b^{7/2} e^2 e^{-\frac{a}{b}} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{a+b \operatorname{ArcCosh}[c+d x]}}{\sqrt{b}}\right]}{128 d} + \frac{35 b^{7/2} e^2 e^{-\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a+b \operatorname{ArcCosh}[c+d x]}}{\sqrt{b}}\right]}{3456 d}
\end{aligned}$$

Result (type 4, 1435 leaves):

$$\begin{aligned}
& \frac{1}{10368 d} e^2 \left( 2592 a^3 c \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + 22680 a b^2 c \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + \right. \\
& 2592 a^3 d x \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + 22680 a b^2 d x \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - 9072 a^2 b \sqrt{\frac{-1+c+d x}{1+c+d x}} \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - \\
& 34020 b^3 \sqrt{\frac{-1+c+d x}{1+c+d x}} \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - 9072 a^2 b c \sqrt{\frac{-1+c+d x}{1+c+d x}} \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - \\
& 34020 b^3 c \sqrt{\frac{-1+c+d x}{1+c+d x}} \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - 9072 a^2 b d x \sqrt{\frac{-1+c+d x}{1+c+d x}} \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - \\
& 34020 b^3 d x \sqrt{\frac{-1+c+d x}{1+c+d x}} \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + 7776 a^2 b c \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + \\
& 22680 b^3 c \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + 7776 a^2 b d x \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + \\
& 22680 b^3 d x \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - 18144 a b^2 \sqrt{\frac{-1+c+d x}{1+c+d x}} \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - \\
& 18144 a b^2 c \sqrt{\frac{-1+c+d x}{1+c+d x}} \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - 18144 a b^2 d x \sqrt{\frac{-1+c+d x}{1+c+d x}} \operatorname{ArcCosh}[c+d x] \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + \\
& \left. 7776 a b^2 c \operatorname{ArcCosh}[c+d x]^2 \sqrt{a+b \operatorname{ArcCosh}[c+d x]} + 7776 a b^2 d x \operatorname{ArcCosh}[c+d x]^2 \sqrt{a+b \operatorname{ArcCosh}[c+d x]} - \right)
\end{aligned}$$

$$\begin{aligned}
& 9072 b^3 \sqrt{\frac{-1+c+dx}{1+c+dx}} \operatorname{ArcCosh}[c+dx]^2 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} - 9072 b^3 c \sqrt{\frac{-1+c+dx}{1+c+dx}} \operatorname{ArcCosh}[c+dx]^2 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} - \\
& 9072 b^3 dx \sqrt{\frac{-1+c+dx}{1+c+dx}} \operatorname{ArcCosh}[c+dx]^2 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} + 2592 b^3 c \operatorname{ArcCosh}[c+dx]^3 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} + \\
& 2592 b^3 dx \operatorname{ArcCosh}[c+dx]^3 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} + 864 a^3 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c+dx]] + \\
& 840 a b^2 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c+dx]] + 2592 a^2 b \operatorname{ArcCosh}[c+dx] \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c+dx]] + \\
& 840 b^3 \operatorname{ArcCosh}[c+dx] \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c+dx]] + \\
& 2592 a b^2 \operatorname{ArcCosh}[c+dx]^2 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c+dx]] + \\
& 864 b^3 \operatorname{ArcCosh}[c+dx]^3 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Cosh}[3 \operatorname{ArcCosh}[c+dx]] + 8505 b^{7/2} \sqrt{\pi} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{Erfi}\left[\frac{\sqrt{a+b \operatorname{ArcCosh}[c+dx]}}{\sqrt{b}}\right] + \\
& 35 b^{7/2} \sqrt{3\pi} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a+b \operatorname{ArcCosh}[c+dx]}}{\sqrt{b}}\right] - 8505 b^{7/2} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{a+b \operatorname{ArcCosh}[c+dx]}}{\sqrt{b}}\right] \operatorname{Sinh}\left[\frac{a}{b}\right] - \\
& 8505 b^{7/2} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{a+b \operatorname{ArcCosh}[c+dx]}}{\sqrt{b}}\right] \left(\operatorname{Cosh}\left[\frac{a}{b}\right] + \operatorname{Sinh}\left[\frac{a}{b}\right]\right) - 35 b^{7/2} \sqrt{3\pi} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a+b \operatorname{ArcCosh}[c+dx]}}{\sqrt{b}}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right] - \\
& 35 b^{7/2} \sqrt{3\pi} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{a+b \operatorname{ArcCosh}[c+dx]}}{\sqrt{b}}\right] \left(\operatorname{Cosh}\left[\frac{3a}{b}\right] + \operatorname{Sinh}\left[\frac{3a}{b}\right]\right) - 1008 a^2 b \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Sinh}[3 \operatorname{ArcCosh}[c+dx]] - \\
& 420 b^3 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Sinh}[3 \operatorname{ArcCosh}[c+dx]] - 2016 a b^2 \operatorname{ArcCosh}[c+dx] \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Sinh}[3 \operatorname{ArcCosh}[c+dx]] - \\
& 1008 b^3 \operatorname{ArcCosh}[c+dx]^2 \sqrt{a+b \operatorname{ArcCosh}[c+dx]} \operatorname{Sinh}[3 \operatorname{ArcCosh}[c+dx]] \Big)
\end{aligned}$$

**Problem 198: Result unnecessarily involves imaginary or complex numbers.**

$$\int (c e + d e x)^{7/2} (a + b \operatorname{ArcCosh}[c + d x]) dx$$

Optimal (type 4, 189 leaves, 8 steps):

$$\begin{aligned}
& -\frac{28 b e^2 \sqrt{-1+c+dx} (e(c+dx))^{3/2} \sqrt{1+c+dx}}{405 d} - \frac{4 b \sqrt{-1+c+dx} (e(c+dx))^{7/2} \sqrt{1+c+dx}}{81 d} + \\
& \frac{2 (e(c+dx))^{9/2} (a+b \operatorname{ArcCosh}[c+dx])}{9 d e} - \frac{28 b e^3 \sqrt{1-c-dx} \sqrt{e(c+dx)} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1+c+dx}}{\sqrt{2}}\right], 2\right]}{135 d \sqrt{-c-dx} \sqrt{-1+c+dx}}
\end{aligned}$$

Result (type 4, 219 leaves):



$$\frac{1}{135 d} (e (c + d x))^{7/2} \left( 30 a (c + d x) - \frac{28 b}{\sqrt{-1 + c + d x} (c + d x)^{5/2} \sqrt{\frac{c + d x}{1 + c + d x}}} - \frac{4 b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} (7 + 5 c^2 + 10 c d x + 5 d^2 x^2)}{3 (c + d x)^2} + \right. \\ \left. 30 b (c + d x) \operatorname{ArcCosh}[c + d x] - \frac{28 i b \sqrt{\frac{c + d x}{1 + c + d x}} \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{(c + d x)^{7/2} \sqrt{\frac{c + d x}{-1 + c + d x}}} \right)$$

**Problem 199: Result unnecessarily involves imaginary or complex numbers.**

$$\int (c e + d e x)^{5/2} (a + b \operatorname{ArcCosh}[c + d x]) dx$$

Optimal (type 4, 169 leaves, 8 steps):

$$-\frac{20 b e^2 \sqrt{-1 + c + d x} \sqrt{e (c + d x)} \sqrt{1 + c + d x}}{147 d} - \frac{4 b \sqrt{-1 + c + d x} (e (c + d x))^{5/2} \sqrt{1 + c + d x}}{49 d} + \\ \frac{2 (e (c + d x))^{7/2} (a + b \operatorname{ArcCosh}[c + d x])}{7 d e} - \frac{20 b e^{5/2} \sqrt{1 - c - d x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], -1\right]}{147 d \sqrt{-1 + c + d x}}$$

Result (type 4, 164 leaves):

$$\frac{1}{147 d (c + d x)^2} (e (c + d x))^{5/2} \left( 21 a (c + d x)^3 - 2 b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} (5 + 3 c^2 + 6 c d x + 3 d^2 x^2) + \right. \\ \left. 21 b (c + d x)^3 \operatorname{ArcCosh}[c + d x] - \frac{10 i b \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{\sqrt{\frac{c + d x}{-1 + c + d x}} \sqrt{1 + c + d x}} \right)$$

**Problem 200: Result unnecessarily involves imaginary or complex numbers.**

$$\int (c e + d e x)^{3/2} (a + b \operatorname{ArcCosh}[c + d x]) dx$$

Optimal (type 4, 145 leaves, 6 steps):

$$-\frac{4b\sqrt{-1+cx}\left(e(cx)\right)^{3/2}\sqrt{1+cx}}{25d} + \frac{2\left(e(cx)\right)^{5/2}\left(a+b\operatorname{ArcCosh}[cx]\right)}{5de} - \frac{12be\sqrt{1-cx}\sqrt{e(cx)}\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1+cx}}{\sqrt{2}}\right], 2\right]}{25d\sqrt{-cx}\sqrt{-1+cx}}$$

Result (type 4, 190 leaves):

$$\frac{1}{25d}2\left(e(cx)\right)^{3/2}\left(5a(cx) - \frac{6b}{\sqrt{-1+cx}\sqrt{cx}\sqrt{\frac{cx}{1+cx}}} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx} + 5b(cx)\operatorname{ArcCosh}[cx] - \frac{6ib\sqrt{\frac{cx}{1+cx}}\sqrt{\frac{1+cx}{-1+cx}}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+cx}}\right], 2\right]}{(cx)^{3/2}\sqrt{\frac{cx}{-1+cx}}}\right)$$

**Problem 201: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{cex}\left(a+b\operatorname{ArcCosh}[cx]\right) dx$$

Optimal (type 4, 127 leaves, 6 steps):

$$-\frac{4b\sqrt{-1+cx}\sqrt{e(cx)}\sqrt{1+cx}}{9d} + \frac{2\left(e(cx)\right)^{3/2}\left(a+b\operatorname{ArcCosh}[cx]\right)}{3de} - \frac{4b\sqrt{e}\sqrt{1-cx}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{e(cx)}}{\sqrt{e}}\right], -1\right]}{9d\sqrt{-1+cx}}$$

Result (type 4, 133 leaves):

$$\frac{1}{9d}2\sqrt{e(cx)}\left(3a(cx) - 2b\sqrt{-1+cx}\sqrt{1+cx} + 3b(cx)\operatorname{ArcCosh}[cx] - \frac{2ib\sqrt{\frac{1+cx}{-1+cx}}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+cx}}\right], 2\right]}{\sqrt{\frac{cx}{-1+cx}}\sqrt{1+cx}}\right)$$

**Problem 202: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[c + d x]}{\sqrt{c e + d e x}} dx$$

Optimal (type 4, 104 leaves, 4 steps):

$$\frac{2 \sqrt{e(c+dx)} (a + b \operatorname{ArcCosh}[c + d x])}{de} - \frac{4 b \sqrt{1-c-dx} \sqrt{e(c+dx)} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1+c+dx}}{\sqrt{2}}\right], 2\right]}{de \sqrt{-c-dx} \sqrt{-1+c+dx}}$$

Result (type 4, 163 leaves):

$$\frac{1}{d \sqrt{e(c+dx)}} \left( 2 \left( a(c+dx) - \frac{2b(c+dx)^{3/2}}{\sqrt{-1+c+dx} \sqrt{\frac{c+dx}{1+c+dx}}} + b(c+dx) \operatorname{ArcCosh}[c + d x] - \frac{2 i b \sqrt{c+dx} \sqrt{\frac{c+dx}{1+c+dx}} \sqrt{\frac{1+c+dx}{-1+c+dx}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+c+dx}}\right], 2\right]}{\sqrt{\frac{c+dx}{-1+c+dx}}} \right) \right)$$

**Problem 203: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcCosh}[c + d x]}{(c e + d e x)^{3/2}} dx$$

Optimal (type 4, 84 leaves, 4 steps):

$$-\frac{2(a + b \operatorname{ArcCosh}[c + d x])}{de \sqrt{e(c+dx)}} + \frac{4b \sqrt{1-c-dx} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right], -1\right]}{de^{3/2} \sqrt{-1+c+dx}}$$

Result (type 4, 115 leaves):

$$\frac{-2 \sqrt{1+c+dx} (a + b \operatorname{ArcCosh}[c + d x]) + \frac{4 i b (c+dx) \sqrt{\frac{1+c+dx}{-1+c+dx}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+c+dx}}\right], 2\right]}{\sqrt{\frac{c+dx}{-1+c+dx}}}}{de \sqrt{e(c+dx)} \sqrt{1+c+dx}}$$

### Problem 204: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c + d x]}{(c e + d e x)^{5/2}} dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\frac{4 b \sqrt{-1 + c + d x} \sqrt{1 + c + d x}}{3 d e^2 \sqrt{e (c + d x)}} - \frac{2 (a + b \operatorname{ArcCosh}[c + d x])}{3 d e (e (c + d x))^{3/2}} - \frac{4 b \sqrt{1 - c - d x} \sqrt{e (c + d x)} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 + c + d x}}{\sqrt{2}}\right], 2\right]}{3 d e^3 \sqrt{-c - d x} \sqrt{-1 + c + d x}}$$

Result (type 4, 197 leaves):

$$\frac{1}{3 d (e (c + d x))^{5/2}} 2 \left( -a (c + d x) - \frac{2 b (c + d x)^{7/2}}{\sqrt{-1 + c + d x} \sqrt{\frac{c + d x}{1 + c + d x}}} + 2 b \sqrt{-1 + c + d x} (c + d x)^2 \sqrt{1 + c + d x} - \right. \\ \left. b (c + d x) \operatorname{ArcCosh}[c + d x] - \frac{2 i b (c + d x)^{5/2} \sqrt{\frac{c + d x}{1 + c + d x}} \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{\sqrt{\frac{c + d x}{-1 + c + d x}}} \right)$$

### Problem 205: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c + d x]}{(c e + d e x)^{7/2}} dx$$

Optimal (type 4, 130 leaves, 7 steps):

$$\frac{4 b \sqrt{-1 + c + d x} \sqrt{1 + c + d x}}{15 d e^2 (e (c + d x))^{3/2}} - \frac{2 (a + b \operatorname{ArcCosh}[c + d x])}{5 d e (e (c + d x))^{5/2}} + \frac{4 b \sqrt{1 - c - d x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], -1\right]}{15 d e^{7/2} \sqrt{-1 + c + d x}}$$

Result (type 4, 121 leaves):

$$\frac{1}{15 d e (e (c + d x))^{5/2}} 2 \left( -3 a + 2 b c \sqrt{-1 + c + d x} \sqrt{1 + c + d x} + \right. \\ \left. 2 b d x \sqrt{-1 + c + d x} \sqrt{1 + c + d x} - 3 b \operatorname{ArcCosh}[c + d x] - i \sqrt{2} b (c + d x)^{5/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-1 + c + d x}\right], \frac{1}{2}\right] \right)$$

### Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int (c e + d e x)^{7/2} (a + b \operatorname{ArcCosh}[c + d x])^2 dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{2 (e (c + d x))^{9/2} (a + b \operatorname{ArcCosh}[c + d x])^2}{9 d e} - \frac{8 b \sqrt{1 - c - d x} (e (c + d x))^{11/2} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{11}{4}, \frac{15}{4}, (c + d x)^2\right]}{99 d e^2 \sqrt{-1 + c + d x}} - \frac{16 b^2 (e (c + d x))^{13/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{13}{4}, \frac{13}{4}\right\}, \left\{\frac{15}{4}, \frac{17}{4}\right\}, (c + d x)^2\right]}{1287 d e^3}$$

Result (type 5, 303 leaves):

$$\frac{1}{9 d} (e (c + d x))^{7/2} \left( 2 a^2 (c + d x) + 4 a b (c + d x) \operatorname{ArcCosh}[c + d x] - \frac{8 a b \sqrt{\frac{c + d x}{1 + c + d x}} \left( \frac{21 + 14 (c + d x) + 2 (c + d x)^3 + 5 (c + d x)^5}{\sqrt{-1 + c + d x}} + \frac{21 i \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{\sqrt{\frac{c + d x}{-1 + c + d x}}}\right)}{45 (c + d x)^{7/2}} + \frac{2}{11} b^2 (c + d x) \operatorname{ArcCosh}[c + d x] \left( 11 \operatorname{ArcCosh}[c + d x] + 4 (c + d x) \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{Hypergeometric2F1}\left[1, \frac{13}{4}, \frac{15}{4}, (c + d x)^2\right] \right) - \frac{945 b^2 \pi (c + d x)^3 \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{13}{4}, \frac{13}{4}\right\}, \left\{\frac{15}{4}, \frac{17}{4}\right\}, (c + d x)^2\right]}{512 \sqrt{2} \operatorname{Gamma}\left[\frac{15}{4}\right] \operatorname{Gamma}\left[\frac{17}{4}\right]} \right)$$

### Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c e + d e x)^{5/2} (a + b \operatorname{ArcCosh}[c + d x])^2 dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{2 (e (c + d x))^{7/2} (a + b \operatorname{ArcCosh}[c + d x])^2}{7 d e} - \frac{8 b \sqrt{1 - c - d x} (e (c + d x))^{9/2} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{9}{4}, \frac{13}{4}, (c + d x)^2\right]}{63 d e^2 \sqrt{-1 + c + d x}} - \frac{16 b^2 (e (c + d x))^{11/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{11}{4}, \frac{11}{4}\right\}, \left\{\frac{13}{4}, \frac{15}{4}\right\}, (c + d x)^2\right]}{693 d e^3}$$

Result (type 5, 369 leaves):

$$\frac{1}{6174 d (c + d x)^2} (e (c + d x))^{5/2} \left( 1764 a^2 (c + d x)^3 + 3528 a b (c + d x)^3 \operatorname{ArcCosh}[c + d x] - \frac{1}{\sqrt{1 + c + d x}} \right. \\ \left. 336 a b \left( \sqrt{-1 + c + d x} (5 + 5 (c + d x) + 3 (c + d x)^2 + 3 (c + d x)^3) + \frac{5 i \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{\sqrt{\frac{c + d x}{-1 + c + d x}}} \right) + \right. \\ \left. b^2 \left( 1336 (c + d x) - 1932 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] + 1323 (c + d x) \operatorname{ArcCosh}[c + d x]^2 + 72 \operatorname{Cosh}[3 \operatorname{ArcCosh}[c + d x]] + 441 \right. \right. \\ \left. \left. \operatorname{ArcCosh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcCosh}[c + d x]] + 1680 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, (c + d x)^2\right] - \right. \right. \\ \left. \left. \frac{210 \sqrt{2} \pi (c + d x) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, (c + d x)^2\right]}{\Gamma\left[\frac{5}{4}\right] \Gamma\left[\frac{7}{4}\right]} - 252 \operatorname{ArcCosh}[c + d x] \operatorname{Sinh}[3 \operatorname{ArcCosh}[c + d x]] \right) \right)$$

**Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (c e + d e x)^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^2 dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{2 (e (c + d x))^{5/2} (a + b \operatorname{ArcCosh}[c + d x])^2}{5 d e} - \frac{8 b \sqrt{1 - c - d x} (e (c + d x))^{7/2} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{4}, \frac{11}{4}, (c + d x)^2\right]}{35 d e^2 \sqrt{-1 + c + d x}}$$

$$\frac{16 b^2 (e (c + d x))^{9/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{9}{4}, \frac{9}{4}\right\}, \left\{\frac{11}{4}, \frac{13}{4}\right\}, (c + d x)^2\right]}{315 d e^3}$$

Result (type 5, 326 leaves):

$$\frac{1}{5 d} (e (c + d x))^{3/2} \left( 2 a^2 (c + d x) + 4 a b (c + d x) \operatorname{ArcCosh}[c + d x] + \right.$$

$$\left. \frac{8}{5} a b \left( -\frac{3}{\sqrt{-1 + c + d x} \sqrt{c + d x} \sqrt{\frac{c + d x}{1 + c + d x}}} - \sqrt{-1 + c + d x} \sqrt{1 + c + d x} - \frac{3 i \sqrt{\frac{c + d x}{1 + c + d x}} \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{(c + d x)^{3/2} \sqrt{\frac{c + d x}{-1 + c + d x}}} \right) + \right.$$

$$\left. \frac{2}{7} b^2 (c + d x) \operatorname{ArcCosh}[c + d x] \left( 7 \operatorname{ArcCosh}[c + d x] + 4 (c + d x) \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{Hypergeometric2F1}\left[1, \frac{9}{4}, \frac{11}{4}, (c + d x)^2\right] \right) - \right.$$

$$\left. \frac{15 b^2 \pi (c + d x)^3 \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{9}{4}, \frac{9}{4}\right\}, \left\{\frac{11}{4}, \frac{13}{4}\right\}, (c + d x)^2\right]}{32 \sqrt{2} \operatorname{Gamma}\left[\frac{11}{4}\right] \operatorname{Gamma}\left[\frac{13}{4}\right]} \right)$$

**Problem 209: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcCosh}[c + d x])^2 dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^2}{3 d e} - \frac{8 b \sqrt{1 - c - d x} (e (c + d x))^{5/2} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, (c + d x)^2\right]}{15 d e^2 \sqrt{-1 + c + d x}}$$

$$\frac{16 b^2 (e (c + d x))^{7/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, (c + d x)^2\right]}{105 d e^3}$$

Result (type 5, 298 leaves):

$$\frac{1}{27 d} \sqrt{e (c + d x)}$$

$$\left( \begin{aligned} & 18 a^2 (c + d x) - 24 a b \sqrt{-1 + c + d x} \sqrt{1 + c + d x} + 36 a b (c + d x) \operatorname{ArcCosh}[c + d x] - 24 b^2 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] + \\ & 2 b^2 (c + d x) (8 + 9 \operatorname{ArcCosh}[c + d x]^2) - \frac{24 i a b \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right]}{\sqrt{\frac{c + d x}{-1 + c + d x}} \sqrt{1 + c + d x}} + 24 b^2 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \\ & \operatorname{ArcCosh}[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, (c + d x)^2\right] - \frac{3 \sqrt{2} b^2 \pi (c + d x) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, (c + d x)^2\right]}{\Gamma\left[\frac{5}{4}\right] \Gamma\left[\frac{7}{4}\right]} \end{aligned} \right)$$

**Problem 210: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c + d x])^2}{\sqrt{c e + d e x}} dx$$

Optimal (type 5, 151 leaves, 3 steps):

$$\frac{2 \sqrt{e (c + d x)} (a + b \operatorname{ArcCosh}[c + d x])^2}{d e} - \frac{8 b \sqrt{1 - c - d x} (e (c + d x))^{3/2} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, (c + d x)^2\right]}{3 d e^2 \sqrt{-1 + c + d x}}$$

$$\frac{16 b^2 (e (c + d x))^{5/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{5}{4}, \frac{5}{4}\right\}, \left\{\frac{7}{4}, \frac{9}{4}\right\}, (c + d x)^2\right]}{15 d e^3}$$

Result (type 5, 268 leaves):



$$\frac{1}{12 d \sqrt{e (c+d x)}} \left( 24 a^2 (c+d x) + 48 a b \left( (c+d x) \operatorname{ArcCosh}[c+d x] - \frac{1}{\sqrt{-1+c+d x} \sqrt{c+d x}} \right. \right. \\ \left. \left. 2 \sqrt{\frac{c+d x}{1+c+d x}} \left( c+d x + (c+d x)^2 + i (-1+c+d x)^{3/2} \sqrt{\frac{c+d x}{-1+c+d x}} \sqrt{\frac{1+c+d x}{-1+c+d x}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+c+d x}}\right], 2\right] \right) \right) + \right. \\ \left. b^2 (c+d x) \left( -\frac{3 \sqrt{2} \pi (c+d x)^2 \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{5}{4}, \frac{5}{4}\right\}, \left\{\frac{7}{4}, \frac{9}{4}\right\}, (c+d x)^2\right]}{\Gamma\left[\frac{7}{4}\right] \Gamma\left[\frac{9}{4}\right]} + \right. \right. \\ \left. \left. 8 \operatorname{ArcCosh}[c+d x] \left( 3 \operatorname{ArcCosh}[c+d x] + 2 \operatorname{Hypergeometric2F1}\left[1, \frac{5}{4}, \frac{7}{4}, (c+d x)^2\right] \operatorname{Sinh}[2 \operatorname{ArcCosh}[c+d x]] \right) \right) \right) \right)$$

**Problem 211: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c + d x])^2}{(c e + d e x)^{3/2}} dx$$

Optimal (type 5, 149 leaves, 3 steps):

$$-\frac{2 (a + b \operatorname{ArcCosh}[c + d x])^2}{d e \sqrt{e (c + d x)}} + \frac{8 b \sqrt{1 - c - d x} \sqrt{e (c + d x)} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, (c + d x)^2\right]}{d e^2 \sqrt{-1 + c + d x}} + \\ \frac{16 b^2 (e (c + d x))^{3/2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, (c + d x)^2\right]}{3 d e^3}$$

Result (type 5, 208 leaves):

$$\frac{1}{d e \sqrt{e(c+dx)}} \left( \frac{8 i a b \sqrt{c+dx} \sqrt{\frac{c+dx}{1+c+dx}} \sqrt{\frac{1+c+dx}{-1+c+dx}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+c+dx}}\right], 2\right] + \frac{\sqrt{2} b^2 \pi (c+dx)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, (c+dx)^2\right]}{\Gamma\left[\frac{5}{4}\right] \Gamma\left[\frac{7}{4}\right]}}{\sqrt{\frac{c+dx}{-1+c+dx}}}\right) - 2 \left( (a+b \operatorname{ArcCosh}[c+dx])^2 + 2 b^2 \operatorname{ArcCosh}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, (c+dx)^2\right] \operatorname{Sinh}[2 \operatorname{ArcCosh}[c+dx]] \right)$$

**Problem 212:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[c+dx])^2}{(c e + d e x)^{5/2}} dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{2(a+b \operatorname{ArcCosh}[c+dx])^2}{3 d e (e(c+dx))^{3/2}} - \frac{8 b \sqrt{1-c-dx} (a+b \operatorname{ArcCosh}[c+dx]) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, (c+dx)^2\right]}{3 d e^2 \sqrt{-1+c+dx} \sqrt{e(c+dx)}} - \frac{16 b^2 \sqrt{e(c+dx)} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{4}, 1\right\}, \left\{\frac{3}{4}, \frac{5}{4}\right\}, (c+dx)^2\right]}{3 d e^3}$$

Result (type 5, 347 leaves):

$$\frac{1}{3 d (e (c + d x))^{5/2}} \left( -2 a^2 (c + d x) - 16 b^2 (c + d x)^3 - 4 a b (c + d x) \operatorname{ArcCosh}[c + d x] + \right.$$

$$8 b^2 (c + d x)^2 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] - 2 b^2 (c + d x) \operatorname{ArcCosh}[c + d x]^2 - \frac{1}{\sqrt{-1 + c + d x}}$$

$$8 a b (c + d x)^{3/2} \sqrt{\frac{c + d x}{1 + c + d x}} \left( 1 + c + d x + i (-1 + c + d x)^{3/2} \sqrt{\frac{c + d x}{-1 + c + d x}} \sqrt{\frac{1 + c + d x}{-1 + c + d x}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1 + c + d x}}\right], 2\right] \right) +$$

$$\frac{8}{3} b^2 (c + d x)^4 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] \operatorname{Hypergeometric2F1}\left[1, \frac{5}{4}, \frac{7}{4}, (c + d x)^2\right] -$$

$$\left. \frac{b^2 \pi (c + d x)^5 \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{5}{4}, \frac{5}{4}\right\}, \left\{\frac{7}{4}, \frac{9}{4}\right\}, (c + d x)^2\right]}{2 \sqrt{2} \operatorname{Gamma}\left[\frac{7}{4}\right] \operatorname{Gamma}\left[\frac{9}{4}\right]}\right)$$

**Problem 213: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{ArcCosh}[c + d x])^2}{(c e + d e x)^{7/2}} dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{2 (a + b \operatorname{ArcCosh}[c + d x])^2}{5 d e (e (c + d x))^{5/2}} - \frac{8 b \sqrt{1 - c - d x} (a + b \operatorname{ArcCosh}[c + d x]) \operatorname{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, (c + d x)^2\right]}{15 d e^2 \sqrt{-1 + c + d x} (e (c + d x))^{3/2}} +$$

$$\frac{16 b^2 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{4}, -\frac{1}{4}, 1\right\}, \left\{\frac{1}{4}, \frac{3}{4}\right\}, (c + d x)^2\right]}{15 d e^3 \sqrt{e (c + d x)}}$$

Result (type 5, 272 leaves):

$$\frac{1}{15 d e (e (c + d x))^{5/2}} \left( -6 a^2 + 4 a b \left( -3 \operatorname{ArcCosh}[c + d x] + (c + d x) \left( 2 \sqrt{-1 + c + d x} \sqrt{1 + c + d x} - i \sqrt{2} (c + d x)^{3/2} \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-1 + c + d x} \right], \frac{1}{2} \right] \right) \right) + b^2 \left( 16 (c + d x)^2 + 8 (c + d x) \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] - 6 \operatorname{ArcCosh}[c + d x]^2 - 8 (c + d x)^3 \sqrt{\frac{-1 + c + d x}{1 + c + d x}} (1 + c + d x) \operatorname{ArcCosh}[c + d x] \operatorname{Hypergeometric2F1}\left[ \frac{3}{4}, 1, \frac{5}{4}, (c + d x)^2 \right] + \frac{\sqrt{2} \pi (c + d x)^4 \operatorname{HypergeometricPFQ}\left[ \left\{ \frac{3}{4}, \frac{3}{4}, 1 \right\}, \left\{ \frac{5}{4}, \frac{7}{4} \right\}, (c + d x)^2 \right]}{\Gamma\left[ \frac{5}{4} \right] \Gamma\left[ \frac{7}{4} \right]} \right) \right)$$

**Problem 214: Attempted integration timed out after 120 seconds.**

$$\int (c e + d e x)^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^3 dx$$

Optimal (type 9, 88 leaves, 2 steps):

$$\frac{2 (e (c + d x))^{5/2} (a + b \operatorname{ArcCosh}[c + d x])^3}{5 d e} - \frac{6 b \operatorname{Unintegrable}\left[ \frac{(e (c + d x))^{5/2} (a + b \operatorname{ArcCosh}[c + d x])^2}{\sqrt{-1 + c + d x} \sqrt{1 + c + d x}}, x \right]}{5 e}$$

Result (type 1, 1 leaves):

???

**Problem 215: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcCosh}[c + d x])^3 dx$$

Optimal (type 9, 86 leaves, 2 steps):

$$\frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^3}{3 d e} - \frac{2 b \operatorname{Unintegrable}\left[ \frac{(e (c + d x))^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^2}{\sqrt{-1 + c + d x} \sqrt{1 + c + d x}}, x \right]}{e}$$

Result (type 1, 1 leaves):

???

Problem 219: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcCosh}[c + d x])^3}{(c e + d e x)^{7/2}} dx$$

Optimal (type 9, 88 leaves, 2 steps):

$$-\frac{2 (a + b \operatorname{ArcCosh}[c + d x])^3}{5 d e (e (c + d x))^{5/2}} + \frac{6 b \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c + d x])^2}{\sqrt{-1 + c + d x} (e (c + d x))^{5/2} \sqrt{1 + c + d x}}, x\right]}{5 e}$$

Result (type 1, 1 leaves):

???

Problem 221: Attempted integration timed out after 120 seconds.

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcCosh}[c + d x])^4 dx$$

Optimal (type 9, 88 leaves, 2 steps):

$$\frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^4}{3 d e} - \frac{8 b \operatorname{Unintegrable}\left[\frac{(e (c + d x))^{3/2} (a + b \operatorname{ArcCosh}[c + d x])^3}{\sqrt{-1 + c + d x} \sqrt{1 + c + d x}}, x\right]}{3 e}$$

Result (type 1, 1 leaves):

???

Problem 225: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcCosh}[c + d x])^4}{(c e + d e x)^{7/2}} dx$$

Optimal (type 9, 88 leaves, 2 steps):

$$-\frac{2 (a + b \operatorname{ArcCosh}[c + d x])^4}{5 d e (e (c + d x))^{5/2}} + \frac{8 b \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c + d x])^3}{\sqrt{-1 + c + d x} (e (c + d x))^{5/2} \sqrt{1 + c + d x}}, x\right]}{5 e}$$

Result (type 1, 1 leaves):

???

### Problem 228: Unable to integrate problem.

$$\int (c e + d e x)^m (a + b \operatorname{ArcCosh}[c + d x])^2 dx$$

Optimal (type 5, 206 leaves, 3 steps):

$$\frac{(e(c+dx))^{1+m} (a+b \operatorname{ArcCosh}[c+dx])^2}{d e (1+m)} - \frac{2 b \sqrt{1-c-dx} (e(c+dx))^{2+m} (a+b \operatorname{ArcCosh}[c+dx]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, (c+dx)^2\right]}{d e^2 (1+m) (2+m) \sqrt{-1+c+dx}}$$

$$\frac{2 b^2 (e(c+dx))^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right\}, (c+dx)^2\right]}{d e^3 (1+m) (2+m) (3+m)}$$

Result (type 8, 25 leaves):

$$\int (c e + d e x)^m (a + b \operatorname{ArcCosh}[c + d x])^2 dx$$

### Problem 229: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (c e + d e x)^m (a + b \operatorname{ArcCosh}[c + d x]) dx$$

Optimal (type 5, 118 leaves, 5 steps):

$$\frac{(e(c+dx))^{1+m} (a+b \operatorname{ArcCosh}[c+dx])}{d e (1+m)} - \frac{b (e(c+dx))^{2+m} (1-(c+dx)^2) \operatorname{Hypergeometric2F1}\left[1, \frac{3+m}{2}, \frac{4+m}{2}, (c+dx)^2\right]}{d e^2 (1+m) (2+m) \sqrt{-1+c+dx} \sqrt{1+c+dx}}$$

Result (type 6, 398 leaves):

$$\frac{1}{d(1+m)} (e(c+dx))^m \left( - \left( \left( 12b \sqrt{-1+c+dx} \sqrt{1+c+dx} \operatorname{AppellF1} \left[ \frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] \right) / \right. \right. \\ \left. \left( 6 \operatorname{AppellF1} \left[ \frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] + \right. \right. \\ \left. \left. (-1+c+dx) \left( 4m \operatorname{AppellF1} \left[ \frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] + \operatorname{AppellF1} \left[ \frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] \right) \right) \right) + \\ \left( 12b \sqrt{\frac{-1+c+dx}{1+c+dx}} \operatorname{AppellF1} \left[ \frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] \right) / \left( 6 \operatorname{AppellF1} \left[ \frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] + \right. \\ \left. (-1+c+dx) \left( 4m \operatorname{AppellF1} \left[ \frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] - \operatorname{AppellF1} \left[ \frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-c-dx, \frac{1}{2}(1-c-dx) \right] \right) \right) + (c+ \\ dx) (a+b \operatorname{ArcCosh}[c+dx]) \right)$$

**Problem 239: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{ArcCosh}[ax^n]}{x} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$-\frac{\operatorname{ArcCosh}[ax^n]^2}{2n} + \frac{\operatorname{ArcCosh}[ax^n] \operatorname{Log}[1 + e^{2 \operatorname{ArcCosh}[ax^n]}]}{n} + \frac{\operatorname{PolyLog}[2, -e^{2 \operatorname{ArcCosh}[ax^n]}]}{2n}$$

Result (type 4, 179 leaves):

$$\operatorname{ArcCosh}[ax^n] \operatorname{Log}[x] +$$

$$\left( a \sqrt{1-a^2 x^{2n}} \left( \operatorname{ArcSinh}[\sqrt{-a^2} x^n]^2 + 2 \operatorname{ArcSinh}[\sqrt{-a^2} x^n] \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[\sqrt{-a^2} x^n]}] \right) - 2n \operatorname{Log}[x] \operatorname{Log}[\sqrt{-a^2} x^n + \sqrt{1-a^2 x^{2n}}] - \right. \\ \left. \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[\sqrt{-a^2} x^n]}] \right) / \left( 2 \sqrt{-a^2} n \sqrt{-1+ax^n} \sqrt{1+ax^n} \right)$$

**Problem 269: Unable to integrate problem.**

$$\int \frac{\left( a + b \operatorname{ArcCosh} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^3}{1-c^2 x^2} dx$$

Optimal (type 4, 265 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^4}{4bc} - \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \frac{3b \left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c} \\
& + \frac{3b^2 \left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c} + \frac{3b^3 \operatorname{PolyLog}\left[4, -e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{4c}
\end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Problem 270: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 196 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{3bc} - \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \\
& + \frac{b \left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \frac{b^2 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c}
\end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Problem 271: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}{1 - c^2 x^2} dx$$

Optimal (type 4, 133 leaves, 6 steps):



$$-\frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{2bc} - \frac{\left(a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c}$$

Result (type 8, 40 leaves):

$$\int \frac{a + b \operatorname{ArcCosh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}{1 - c^2 x^2} dx$$

Problem 274: Attempted integration timed out after 120 seconds.

$$\int \operatorname{ArcCosh}\left[c e^{a+bx}\right] dx$$

Optimal (type 4, 76 leaves, 6 steps):

$$-\frac{\operatorname{ArcCosh}\left[c e^{a+bx}\right]^2}{2b} + \frac{\operatorname{ArcCosh}\left[c e^{a+bx}\right] \operatorname{Log}\left[1 + e^{2 \operatorname{ArcCosh}\left[c e^{a+bx}\right]}\right]}{b} + \frac{\operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}\left[c e^{a+bx}\right]}\right]}{2b}$$

Result (type 1, 1 leaves):

???

Problem 278: Result more than twice size of optimal antiderivative.

$$\int e^{\operatorname{ArcCosh}[a+bx]} dx$$

Optimal (type 3, 31 leaves, 5 steps):

$$\frac{e^{2 \operatorname{ArcCosh}[a+bx]}}{4b} - \frac{\operatorname{ArcCosh}[a+bx]}{2b}$$

Result (type 3, 69 leaves):

$$\frac{(a+bx) \left(a+bx + \sqrt{-1+a+bx} \sqrt{1+a+bx}\right) - \operatorname{Log}\left[a+bx + \sqrt{-1+a+bx} \sqrt{1+a+bx}\right]}{2b}$$

Problem 279: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\operatorname{ArcCosh}[a+bx]}}{x} dx$$

Optimal (type 3, 100 leaves, 9 steps):

$$b x + \sqrt{-1+a+b x} \sqrt{1+a+b x} + 2 a \operatorname{ArcSinh}\left[\frac{\sqrt{-1+a+b x}}{\sqrt{2}}\right] + 2 \sqrt{1-a^2} \operatorname{ArcTan}\left[\frac{\sqrt{1-a} \sqrt{1+a+b x}}{\sqrt{1+a} \sqrt{-1+a+b x}}\right] + a \operatorname{Log}[x]$$

Result (type 3, 141 leaves):

$$b x + \sqrt{-1+a+b x} \sqrt{1+a+b x} + a \operatorname{Log}[x] + a \operatorname{Log}\left[a+b x + \sqrt{-1+a+b x} \sqrt{1+a+b x}\right] + i \sqrt{1-a^2} \operatorname{Log}\left[\frac{2 \sqrt{-1+a+b x} \sqrt{1+a+b x}}{(-1+a^2) x} + \frac{2 i (-1+a^2+a b x)}{\sqrt{1-a^2} (-1+a^2) x}\right]$$

Problem 280: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\operatorname{ArcCosh}[a+b x]}}{x^2} dx$$

Optimal (type 3, 109 leaves, 9 steps):

$$-\frac{a}{x} - \frac{\sqrt{-1+a+b x} \sqrt{1+a+b x}}{x} + 2 b \operatorname{ArcSinh}\left[\frac{\sqrt{-1+a+b x}}{\sqrt{2}}\right] - \frac{2 a b \operatorname{ArcTan}\left[\frac{\sqrt{1-a} \sqrt{1+a+b x}}{\sqrt{1+a} \sqrt{-1+a+b x}}\right]}{\sqrt{1-a^2}} + b \operatorname{Log}[x]$$

Result (type 3, 140 leaves):

$$-\frac{a}{x} - \frac{\sqrt{-1+a+b x} \sqrt{1+a+b x}}{x} + b \operatorname{Log}[x] + b \operatorname{Log}\left[a+b x + \sqrt{-1+a+b x} \sqrt{1+a+b x}\right] - \frac{i a b \operatorname{Log}\left[\frac{2\left(\sqrt{-1+a+b x} \sqrt{1+a+b x} + \frac{i(-1+a^2+a b x)}{\sqrt{1-a^2}}\right)}{a b x}\right]}{\sqrt{1-a^2}}$$

Problem 281: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\operatorname{ArcCosh}[a+b x]}}{x^3} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$-\frac{a}{2 x^2} - \frac{b}{x} + \frac{b \sqrt{-1+a+b x} \sqrt{1+a+b x}}{2(1-a^2) x} - \frac{\sqrt{-1+a+b x} (1+a+b x)^{3/2}}{2(1+a) x^2} - \frac{b^2 \operatorname{ArcTan}\left[\frac{\sqrt{1-a} \sqrt{1+a+b x}}{\sqrt{1+a} \sqrt{-1+a+b x}}\right]}{(1-a^2)^{3/2}}$$

Result (type 3, 142 leaves):

$$\frac{1}{2} \left( -\frac{a}{x^2} - \frac{2 b}{x} - \frac{\sqrt{-1+a+b x} \sqrt{1+a+b x} (-1+a^2+a b x)}{(-1+a^2) x^2} - \frac{i b^2 \operatorname{Log}\left[\frac{4 i \sqrt{1-a^2} (-1+a^2+a b x - i \sqrt{1-a^2} \sqrt{-1+a+b x} \sqrt{1+a+b x})}{b^2 x}\right]}{(1-a^2)^{3/2}} \right)$$

**Problem 282:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcCosh}[a+bx]}}{x^4} dx$$

Optimal (type 3, 189 leaves, 8 steps):

$$-\frac{a}{3x^3} - \frac{b}{2x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2} + \frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3(1-a^2)x^3} - \frac{ab^3\text{ArcTan}\left[\frac{\sqrt{1-a}\sqrt{1+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right]}{(1-a^2)^{5/2}}$$

Result (type 3, 179 leaves):

$$\frac{1}{6} \left( -\frac{2a}{x^3} - \frac{3b}{x^2} + \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}(-2-2a^4+abx-a^3bx+2b^2x^2+a^2(4+b^2x^2))}{(-1+a^2)^2x^3} - \frac{3ia^3\text{Log}\left[\frac{4(1-a^2)^{3/2}(-i+ia^2+iabx+\sqrt{1-a^2}\sqrt{-1+a+bx}\sqrt{1+a+bx})}{ab^3x}\right]}{(1-a^2)^{5/2}} \right)$$

**Problem 283:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcCosh}[a+bx]}}{x^5} dx$$

Optimal (type 3, 238 leaves, 10 steps):

$$-\frac{a}{4x^4} - \frac{b}{3x^3} - \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4} + \frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12(1-a^2)x^3} + \frac{(3+2a^2)b^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{24(1-a^2)^2x^2} + \frac{a(13+2a^2)b^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}{24(1-a^2)^3x} - \frac{(1+4a^2)b^4\text{ArcTan}\left[\frac{\sqrt{1-a}\sqrt{1+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right]}{4(1-a^2)^{7/2}}$$

Result (type 3, 198 leaves):

$$\frac{1}{24} \left( -\frac{6a}{x^4} - \frac{8b}{x^3} - \frac{\sqrt{-1+a+bx} \sqrt{1+a+bx} \left( 6 + \frac{2abx}{-1+a^2} - \frac{(3+2a^2)b^2x^2}{(-1+a^2)^2} + \frac{a(13+2a^2)b^3x^3}{(-1+a^2)^3} \right)}{x^4} - \frac{3i(1+4a^2)b^4 \operatorname{Log} \left[ \frac{16i(1-a^2)^{5/2}(-1+a^2+abx-i\sqrt{1-a^2}\sqrt{-1+a+bx}\sqrt{1+a+bx})}{b^4(x+4a^2x)} \right]}{(1-a^2)^{7/2}} \right)$$

**Problem 294:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCosh} \left[ \frac{c}{a+bx} \right] dx$$

Optimal (type 3, 58 leaves, 5 steps):

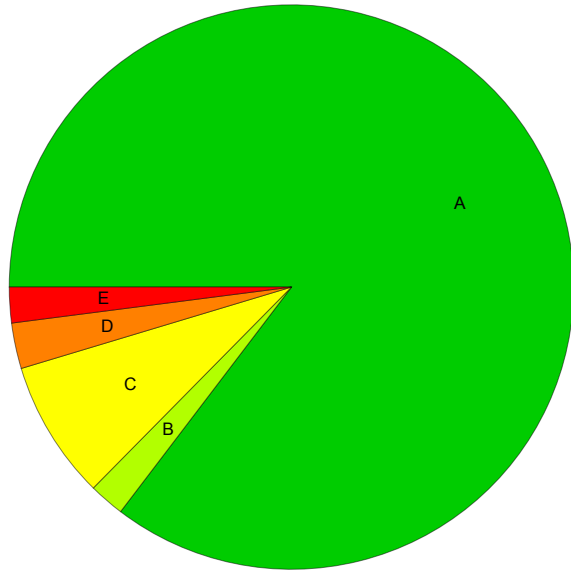
$$\frac{(a+bx) \operatorname{ArcSech} \left[ \frac{a}{c} + \frac{bx}{c} \right]}{b} - \frac{2c \operatorname{ArcTan} \left[ \sqrt{\frac{(1-\frac{a}{c})c-bx}{a+c+bx}} \right]}{b}$$

Result (type 3, 143 leaves):

$$x \operatorname{ArcCosh} \left[ \frac{c}{a+bx} \right] + \frac{\sqrt{a-c+bx} \left( i a \operatorname{Log} \left[ -\frac{2b^2(-ic+\sqrt{a-c+bx}\sqrt{a+c+bx})}{a(a+bx)} \right] + c \operatorname{Log} \left[ a+bx+\sqrt{a-c+bx}\sqrt{a+c+bx} \right] \right)}{b \sqrt{-\frac{a-c+bx}{a+c+bx}} \sqrt{a+c+bx}}$$

## Summary of Integration Test Results

1031 integration problems



A - 880 optimal antiderivatives

B - 21 more than twice size of optimal antiderivatives

C - 82 unnecessarily complex antiderivatives

D - 27 unable to integrate problems

E - 21 integration timeouts